ME 326 - Intermediate Fluid Mechanics

Clarkson University

Turbulent Boundary Layer

Goodarz Ahmadi

Department of Mechanical and Aeronautical Engineering
Clarkson University
Potsdam, NY 13699-5725

Turbulent Boundary Layer

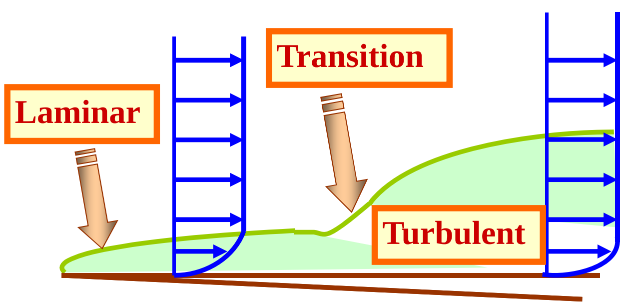
Clarkson University

Outline

- Transition
- Turbulent Boundary Layer Flows
- Momentum Integral Method
- Drag on Smooth Plates
- Drag on Rough Plates
- Empirical Equations for Drag

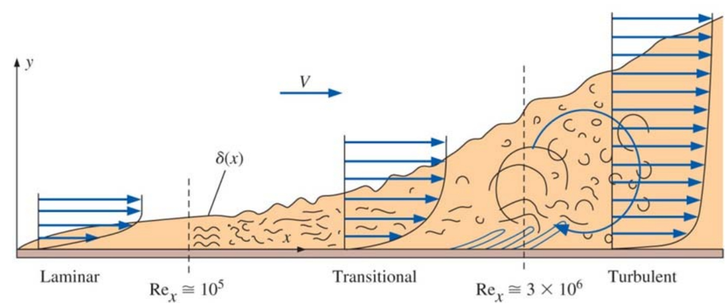
Boundary Layer Transition

Clarkson University



Boundary Layer Transition

Clarkson University



Boundary Layer over a Flat Plate

Clarkson University

Boundary Layer Thickness → Laminar →

$$\frac{\delta}{x} = 5\,\mathrm{Re}_x^{-\frac{1}{2}}$$

Friction Coefficient

$$C_D = 1.328\,\mathrm{Re}_\ell^{-\frac{1}{2}}$$

$$\mathrm{Re}_\ell = \frac{U_0 \ell}{\nu}$$

Critical Reynolds Number →

$$\mathrm{Re}_{crit} \approx \begin{cases} 3.2\times10^5 \sim 10^6 \\ 5\times10^5 \sim 3\times10^6 \end{cases}$$

ME 326 G. Ahmadi

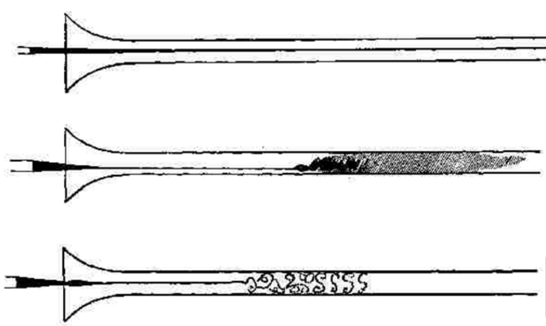
Reynolds' Experiments

Reynolds Pipe Flow Experiment

ME 326 G. Ahmadi

Reynolds' Experiments

Reynolds Pipe Flow Experiment



Laminar

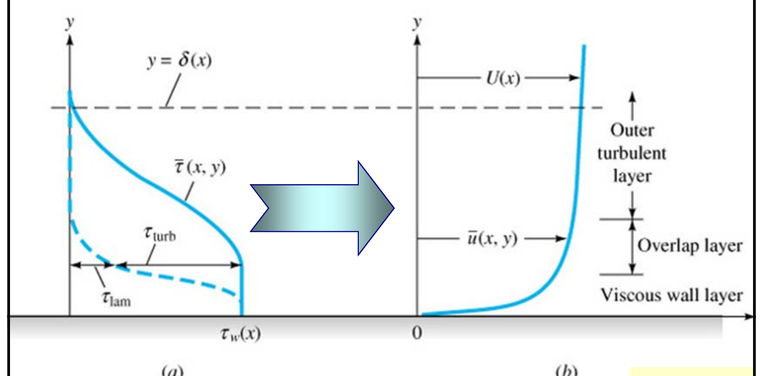
Turbulent

Transition

G. Ahmadi

Turbulent Velocity Profiles

Clarkson University

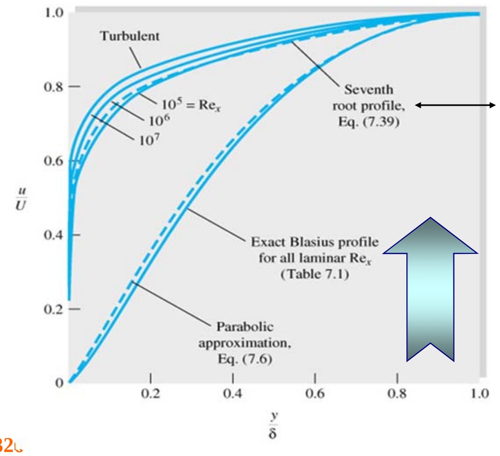


White

ME 326 G. Ahmadi

Laminar and Turbulent Velocity Profiles

Clarkson University



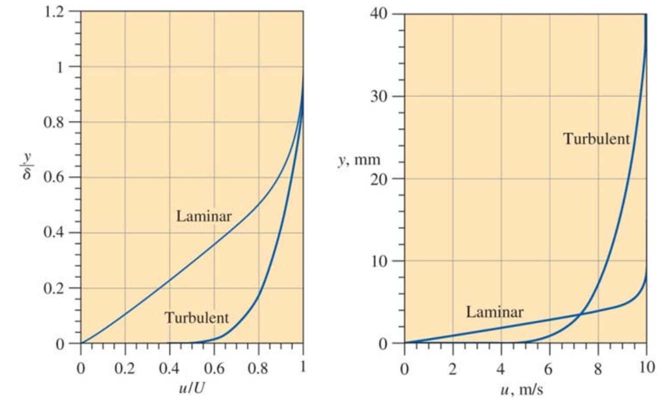
$$\frac{u}{U_o} = \left(\frac{y}{\delta}\right)^{1/7}$$

White

ME 326 G. Ahmadi

Laminar and Turbulent Velocity Profiles

Clarkson University



ME 326 G. Ahmadi

Inertial Sublayer

Clarkson University

Turbulent Stress=Wall Shear Stress

$$\tau_0 = \rho\kappa^2 y^2 \left(\frac{\partial u}{\partial y}\right)^2$$

$$\frac{du}{dy} = \frac{u^*}{\kappa y}$$

$$u^* = \sqrt{\frac{\tau_0}{\rho}}$$

$$\frac{u}{u^*} = u^+ = \frac{1}{\kappa}\ln y + c$$

$$y^+ = \frac{u^* y}{\nu}$$

$$B \approx 5$$

Wall Units

$$u^+ = \frac{1}{\kappa}\ln y^+ + B$$

$$30 < y^+ \le 300$$

ME 326 G. Ahmadi

Viscous Sublayer

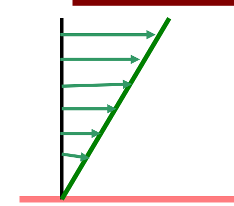
Clarkson University

Turbulent stress is negligible

$$0 < y^+ \le 5$$

$$\tau_0 = \mu \frac{du}{dy}$$

$$u^{*2} = \nu \frac{du}{dy}$$



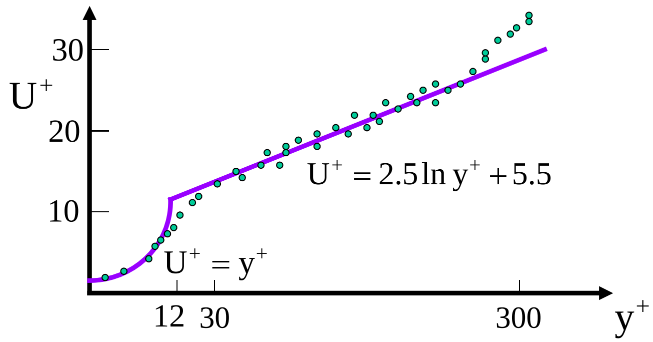
$$\frac{du^+}{dy^+} = 1$$

$$u^+ = y^+$$

ME 326 G. Ahmadi

Velocity Near a Wall

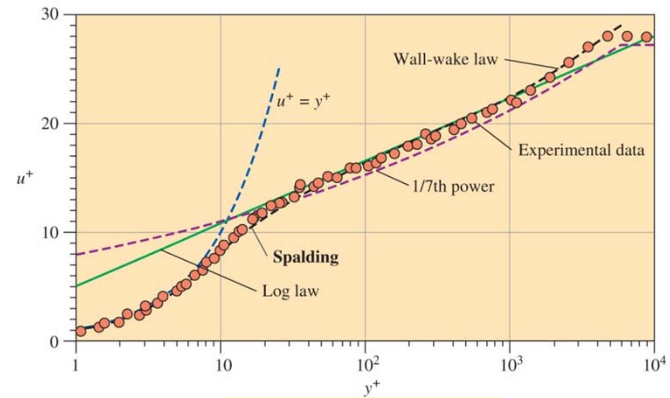
Clarkson University



ME 326 G. Ahmadi

Velocity Near a Wall

Clarkson University



Spalding, Eq. (10-85)

ME 326 G. Ahmadi

Momentum Integral Method

Clarkson University

$$\frac{\tau_0}{\rho U_0^2} = \frac{d\theta}{dx} \quad \Longleftrightarrow \quad C_{f,x} = 2\frac{d\theta}{dx}$$

Prandtl Suggestion → **Turbulence** → $C_{f,x} = 0.02\,\mathrm{Re}_\delta^{-1/6}$

$$\mathrm{Re}_\delta = \frac{U_0 \delta}{\nu}$$

$$\theta = \int_0^\infty \frac{u}{U_0}\left(1 - \frac{u}{U_0}\right) dy$$

$$\frac{u}{U_0} = \left(\frac{y}{\delta}\right)^{1/7} \rightarrow \theta = \int_0^\delta \left(\frac{y}{\delta}\right)^{1/7}\left(1 - \left(\frac{y}{\delta}\right)^{1/7}\right) dy = \frac{7}{72}\delta$$

ME 326 G. Ahmadi

Momentum Integral Method

Clarkson University

$$C_{f,x} = 2\frac{d\theta}{dx} \rightarrow C_{f,x} = 0.02\,\mathrm{Re}_\delta^{-1/6} = \frac{7}{72}\frac{d\delta}{dx}$$

$$\rightarrow \frac{d\delta}{dx} = \frac{1.44}{7}\left(\frac{\nu}{U_0 \delta}\right)^{1/6} \rightarrow \delta^{1/6} d\delta = 0.205\left(\frac{\nu}{U_0}\right)^{1/6} dx$$

$$\rightarrow \delta = 0.16\left(\frac{\nu}{U_0}\right)^{1/7} x^{6/7} \rightarrow \frac{\delta}{x} = \frac{0.16}{\mathrm{Re}_x^{1/7}} \qquad \frac{\theta}{x} = \frac{0.0155}{\mathrm{Re}_x^{1/7}}$$

Turbulent Boundary Layer Thickness

ME 326

Turbulent Boundary Layer -Smooth

Clarkson University

$$\frac{\delta}{x} = \frac{0.16}{Re_x^{1/7}}$$

$$\delta^* = \frac{1}{8}\delta$$

$$H = \frac{\delta^*}{\theta} = 1.3$$

Friction Coefficient

$$C_{f,x} = 2\frac{d\theta}{dx} = \frac{0.027}{Re_x^{1/7}}$$

Drag Coefficient

$$C_D = 2\frac{\theta(L)}{L} = \frac{0.031}{Re_L^{1/7}}$$

ME 326 G. Ahmadi

Transition

Clarkson University

Schlichting Equations

$$C_D = \frac{0.031}{Re_L^{1/7}} - \frac{1440}{Re_L} \qquad Re_{Trans} = 5\times10^5$$

$$C_D = \frac{0.031}{Re_L^{1/7}} - \frac{8700}{Re_L} \qquad Re_{Trans} = 3\times10^6$$

ME 326 G. Ahmadi

Average Friction Coefficient

Clarkson University

Laminar

$$C_D = \frac{1.326}{Re_L^{1/2}} \qquad Re_L \le 5\times10^5$$

Turbulent

$$C_D = \frac{0.074}{Re_L^{1/5}} \qquad 5\times10^5 \le Re_L \le 10^7$$

Corrected for laminar part

$$C_D = \frac{0.074}{Re_L^{1/5}} - \frac{1742}{Re_L} \qquad 5\times10^5 \le Re_L \le 10^7$$

ME 326 G. Ahmadi

Rough Walls

Clarkson University

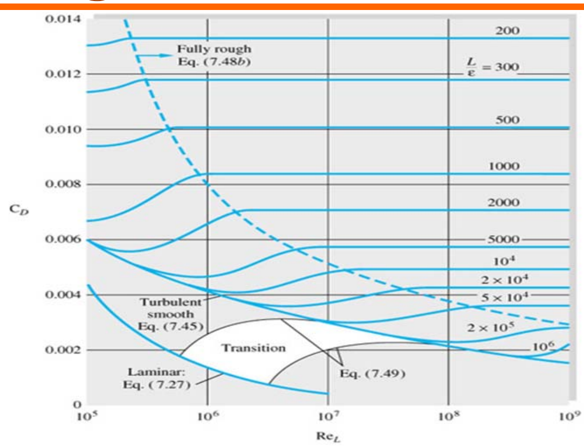
$$C_F = \left(2.87 + 1.58\log\frac{x}{\varepsilon}\right)^{-2.5}$$

$$C_D = \left(1.89 + 1.62\log\frac{L}{\varepsilon}\right)^{-2.5}$$

ME 326 G. Ahmadi

Drag Coefficient

Clarkson University



White

ME 326 G. Ahmadi

Turbulent Boundary Layer

Clarkson University

Concluding Remarks

- Transition
- Turbulent Boundary Layer Flows
- Momentum Integral Method
- Drag on Smooth Plates
- Drag on Rough Plates
- Empirical Equations for Drag

ME 326 G. Ahmadi

Clarkson University

Thank you!

Questions?

ME 326 G. Ahmadi