ME 326 - Intermediate Fluid Mechanics

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Isentropic Flows

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Outline

- Compressible Flow Regimes
  - Thermodynamics
  - Speed of Sound & Mach Number
- Isentropic Flows with Area Change
  - Variations with Mach number
- Shock Waves
  - Nozzle and Diffusers
- Flows with Friction
- Flows with Heat Transfer

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Energy Equation

$$(h_1 + \frac{V_1^2}{2}) = (h_2 + \frac{V_2^2}{2}) = h_o$$

Ideal Gas

$$h_o = (c_p T + \frac{V^2}{2}) = c_p T_o$$

Stagnation Conditions

Temperature Ratio

$$\frac{T_o}{T} = 1 + \frac{V^2}{2c_p T} = 1 + \frac{V^2}{2\frac{kR}{k-1}T} = 1 + \frac{k-1}{2}M^2$$

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Temperature Ratio

$$\frac{T_o}{T} = 1 + \frac{k-1}{2}M^2$$

Pressure Ratio

$$\frac{P_o}{P} = \left(1 + \frac{k-1}{2}M^2\right)^{k/(k-1)}$$

Density Ratio

$$\frac{\rho_o}{\rho} = \left(1 + \frac{k-1}{2}M^2\right)^{1/(k-1)}$$

$$\frac{P_2}{P_1} = \left[\frac{T_2}{T_1}\right]^{\frac{k}{k-1}} = \left[\frac{\rho_2}{\rho_1}\right]^k$$

$$\frac{c_o}{c} = \left(1 + \frac{k-1}{2}M^2\right)^{1/2}$$

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Critical Values at M=1 **k=1.4**

$$\frac{T^*}{T_o} = \frac{2}{k+1} = 0.833$$

$$\frac{P^*}{P_o} = \left(\frac{2}{k+1}\right)^{k/(k-1)} = 0.528$$

$$\frac{\rho^*}{\rho_o} = \left(\frac{2}{k+1}\right)^{1/(k-1)} = 0.634$$

$$\frac{c^*}{c_o} = \left(\frac{2}{k+1}\right)^{1/2} = 0.913$$

$$V^* = c^* = (kRT^*)^{1/2} = \left(\frac{2k}{k+1} RT_o\right)^{1/2} = \left(\frac{2}{k+1} c_o\right)^{1/2}$$

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k=1.4

$$\frac{T_o}{T} = 1 + 0.2M^2$$

$$\frac{P_o}{P} = \left(1 + 0.2M^2\right)^{3.5}$$

$$\frac{\rho_o}{\rho} = \left(1 + 0.2M^2\right)^{2.5}$$

$$\frac{c_o}{c} = \left(1 + 0.2M^2\right)^{1/2}$$

$$M^2 = 5\left(\frac{T_o}{T} - 1\right) = 5\left(\left(\frac{P_o}{P}\right)^{2/7} - 1\right) = 5\left(\left(\frac{\rho_o}{\rho}\right)^{2/5} - 1\right)$$

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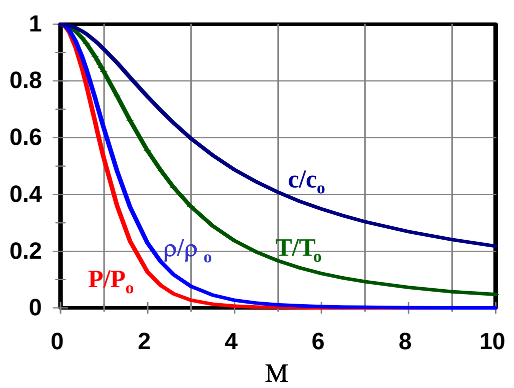

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Isentropic Flows with Area Change

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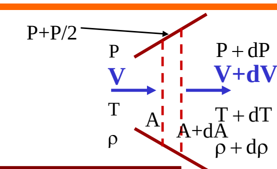

Energy Equation

$$h_o = h + \frac{V^2}{2} \quad \Rightarrow \quad dh = -d\left(\frac{V^2}{2}\right)$$

Continuity Equation

$$\rho VA = \text{Const.} \quad \Rightarrow \quad \frac{d\rho}{\rho} + \frac{dV}{V} + \frac{dA}{A} = 0$$

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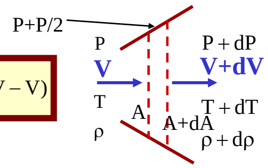
Momentum

$$PA-(P+\frac{dP}{2})dA-(P+dP)(A+dA)=\rho VA(V+dV-V)$$



$$dP=-\rho VdV$$

$$\frac{dA}{A}=-\frac{d\rho}{\rho}-\frac{dV}{V}$$

$$\frac{dA}{A}=\frac{dP}{\rho V^2}(1-M^2)$$

$$\frac{dV}{V}=-\frac{1}{1-M^2}\frac{dA}{A}$$

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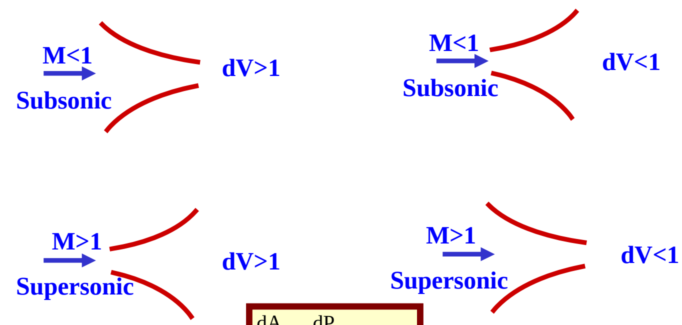
Nozzle, dP<0

Diffuser, dP>0



$$\frac{dA}{A}=\frac{dP}{\rho V^2}(1-M^2)$$

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Area Ratio

$$\rho VA=\rho^* V^* A^*=G^* A^*=GA$$

$$\frac{A}{A^*}=\frac{\rho^* V^*}{\rho V}=\frac{G^*}{G}$$

$$\frac{\rho^*}{\rho}=\frac{\rho^*}{\rho_o}\frac{\rho_o}{\rho}=\left(\frac{1+\frac{k-1}{2}M^2}{\frac{k+1}{2}}\right)^{1/(k-1)}$$

$$\frac{V^*}{V}=\frac{1}{M}\frac{c^*}{c}=\frac{1}{M}\sqrt{\frac{T^*}{T}}$$

$$\frac{V^*}{V}=\frac{1}{M}\left(\frac{1+\frac{k-1}{2}M^2}{\frac{k+1}{2}}\right)^{1/2}$$

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Area Ratio

$$\frac{A}{A^*}=\frac{1}{M}\left(\frac{1+\frac{k-1}{2}M^2}{\frac{k+1}{2}}\right)^{(k+1)/2(k-1)}=\frac{G^*}{G}$$

k=1.4

$$\frac{A}{A^*}=\frac{1}{M}\frac{\left(1+0.2M^2\right)^3}{1.728}$$

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M*

$$M^* = \frac{V}{c^*} = \frac{Mc}{c^*} = \frac{M\sqrt{kRT}}{\sqrt{kRT^*}} = M\sqrt{\frac{T}{T^*}}$$

$$M^* = M\sqrt{\frac{k+1}{2+(k-1)M^2}}$$

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Mass Flow Rate

$$\dot{m} = \rho AV$$

$$\dot{m} = \frac{P}{RT}AM\sqrt{kRT} = APM\sqrt{\frac{k}{RT}}$$

$$\dot{m} = \frac{AMP_o\sqrt{k/(RT_o)}}{\left(1+\frac{k-1}{2}M^2\right)^{(k+1)/[2(k-1)]}}$$

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Maximum Mass Flow Rate

Choking, M=1

$$\dot{m}_{max} = A^* P_o\sqrt{\frac{k}{RT_o}}\left(\frac{2}{k+1}\right)^{(k+1)/[2(k-1)]}$$

$$\dot{m}_{max} = \left(\frac{2}{k+1}\right)^{(k+1)/2(k-1)} A^* \underbrace{\rho_o\sqrt{kRT_o}}_{\sqrt{kP_o\rho_o}}$$

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Maximum Mass Flow Rate

Choking, M=1

k=1.4

$$\dot{m}_{max} = 0.6847A^*\rho_o\sqrt{RT_o} = \frac{0.6847A^*P_o}{\sqrt{RT_o}}$$

$$\dot{m}_{max} = \rho^* A^* V^*$$

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Mass Flow Rate

$$\dot{m} = \rho A V$$

$$\dot{m} = \left(\frac{2k}{k-1} P_o \rho_o \left(\frac{P}{P_o}\right)^{2/k} \left[1 - \left(\frac{P}{P_o}\right)^{(k-1)/k}\right] \right)^{1/2} A$$

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Table 1

M	P/P_o	ρ/ρ_o	T/T_o	A/A^*
0	1	1	1	-
0.4	0.896	0.934	0.969	1.59
1	0.528	0.634	0.833	1
2	0.128	0.23	0.556	1.688
4	0.0066	0.0277	0.238	10.71

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Concluding Remarks

- Isentropic Flows with Area Change
- Variations with Mach Number
- Mass Flow Rate and Choking
- Nozzle and Diffusers

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Thank you!

Questions?

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