ME 639 **Final Exam** **April 27, 2004**

1. (20 Points) Consider the turbulent flow of an incompressible fluid with Pr=1. Estimate the order of magnitude of the following quantities in terms of u, λ, θ and Λ:

$$\text{a) } \overline{\omega'_i\omega'_j \frac{\partial u'_i}{\partial x_k}\frac{\partial u'_j}{\partial x_k}} \qquad \text{b) } \overline{\frac{\partial \omega'_i}{\partial x_j}\frac{\partial \omega'_j}{\partial x_i}} \qquad \text{c) } \overline{\frac{\partial u'_k}{\partial x_j}\frac{\partial \omega'_j}{\partial x_k}}$$

2. (20 Points) Let $\Im = \delta[\mathbf{u}(\mathbf{x},t) - \mathbf{U}]$, where $\mathbf{u}(\mathbf{x},t)$ is the velocity vector of an incompressible fluid and $\mathbf{U}$ is the velocity coordinates. Evaluate the following expressions:

$$\text{a) } < \frac{\partial}{\partial x_j}\left(\frac{\partial \Im}{\partial u_k} u_k\right) > \qquad \text{b) } < x_i u_j \frac{\partial}{\partial x_j}[(1-u_i)\Im] >$$

$$\text{c) } < \frac{\partial^2}{\partial x_i \partial u_j}(u_i x_m U_m \Im) > \qquad \text{d) } < \frac{\partial^2 \Im}{\partial x_j \partial U_k}\frac{\partial u_k}{\partial x_j} + \frac{\partial \Im}{\partial U_k}\frac{\partial^2 u_k}{\partial x_j \partial x_j} >$$

3. (20 Points) Suppose u(t) satisfies the following equation.

$$\frac{du}{dt} + u + u^3 = 0.$$

Find the equation governing the evolution of the probability density function of u.

4. (40 Points) Derive the transport equation for $\overline{\frac{\partial u'_i}{\partial x_j}\frac{\partial u'_i}{\partial x_j}} = \overline{u'_{i,j}u'_{i,j}}$. Identify the terms corresponding to production, dissipation, diffusion and convection. Find the order of magnitude of different terms. Propose closure models for different terms.