ME 639 **Midterm Exam** **April 2004**

1. (20 Points) Consider the turbulent flow of an incompressible fluid with Pr=1. Estimate the order of magnitude of the following quantities in terms of u, λ, θ and Λ:

$$\text{a)}\ \overline{u_k' \frac{\partial u_k'}{\partial x_i} u_i'} \qquad \text{b)}\ \overline{\frac{\partial^2 \omega_i'}{\partial x_j \partial x_j} \frac{\partial \omega_i'}{\partial x_k \partial x_k}} \qquad \text{c)}\ \overline{\omega_i' \frac{\partial u_j'}{\partial x_i} u_j'}$$

$$\text{d)}\ \overline{\omega_i' \omega_j' \frac{\partial u_i'}{\partial x_k} \frac{\partial u_j'}{\partial x_k}} \qquad \text{e)}\ \overline{\frac{\partial u_i'}{\partial x_k} \frac{\partial u_j'}{\partial x_k} D_{ij}} \qquad \text{f)}\ \overline{\frac{\partial \omega_k'}{\partial x_j} \frac{\partial^2 \omega_i'}{\partial x_i \partial x_k}}$$

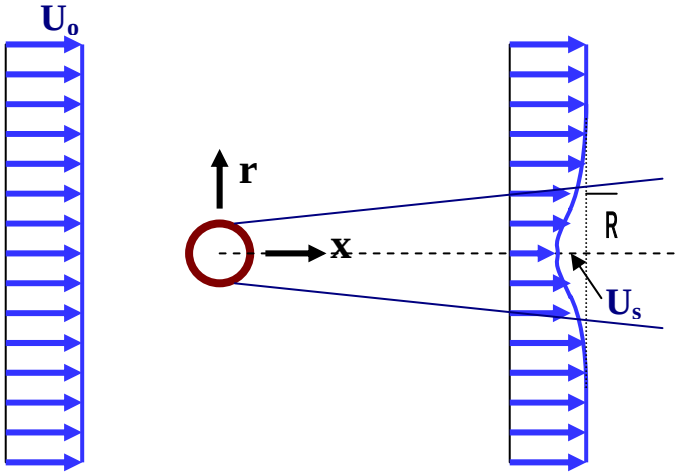
2. (30 Points) The Burger equations for momentum and hear transfer are given as

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \nu \frac{\partial^2 u}{\partial x^2} \quad \text{and} \quad \frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} = \nu \frac{\partial^2 T}{\partial x^2}$$

Derive the transport equation for the velocity-temperature correlation $\overline{u'T'}$ for the burger model. Identify the terms corresponding to production, dissipation, diffusion and convection. Find the order of magnitude of different terms.

3. (25 Points) For a laminar axisymmetric wake flow behind a sphere, obtain the variations of U_s and R with x. The equation governing of motion is given as

$$U_o \frac{\partial U}{\partial x} = \frac{\nu}{r}\frac{\partial}{\partial r}\left(r \frac{\partial U}{\partial r}\right)$$



4. (25 Points) Determine the contribution of eddies of size r to the correlations a) $\overline{\omega_i' \omega_j' \frac{\partial u_i'}{\partial x_k} u_j'}$ b) $\overline{\frac{\partial \omega_k'}{\partial x_j} \frac{\partial \omega_k'}{\partial x_j}}$. For r being Λ, λ, and η evaluate the these contribution and compare.