

Wiener-Hermite Expansions Method

Goodarz Ahmadi

Department of Mechanical and Aeronautical Engineering
Clarkson University
Potsdam, NY 13699-5725

Outline

- Expansion of a function
- Orthogonal set
- Wiener-Hermite orthogonal set
- Expansion of a random function
- Burger Equation
- W-H expansion for the Burger Eq.

Let $\varphi_n(x)$ be an orthogonal set 

$$\int \varphi_n \varphi_m dx = \delta_{nm} \|\varphi_n\|$$

$$\|\varphi_n\| = \int \varphi_n^2 dx$$

Let $u(x)$ be an arbitrary function

$$u(x) = \sum_n c_n \varphi_n(x)$$

$$c_n = \frac{\int u \varphi_n dx}{\|\varphi_n\|}$$

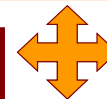
Let $a(x)$ be a white noise process 

$$\langle a(x) \rangle = 0$$

$$\langle a(x_1) a(x_2) \rangle = \delta(x_1 - x_2)$$

Wiener -Hermite base

$$H^{(0)}(x) = 1$$



$$H^{(1)}(x) = a(x)$$

$$H^{(2)}(x_1, x_2) = a(x_1) a(x_2) - \delta(x_1 - x_2)$$

Wiener-Hermite Expansions Clarkson University

Wiener -Hermite base

$$H^{(3)}(x_1, x_2, x_3) = a(x_1)a(x_2)a(x_3) - a(x_1)\delta(x_2 - x_3) - a(x_2)\delta(x_3 - x_1) - a(x_3)\delta(x_1 - x_2)$$

$$\langle H^{(i)} H^{(j)} \rangle = 0 \quad i \neq j$$

Wiener -Hermite set is complete

ME 639-Turbulence

G. Ahmadi

Wiener-Hermite Expansions Clarkson University

Wiener -Hermite base

$$\langle H^{(0)}(x) H^{(0)}(x) \rangle = 1$$

$$\langle H^{(1)}(x_1) H^{(1)}(x_2) \rangle = \delta(x_1 - x_2)$$

$$\langle H^{(2)}(x_1, x_2) H^{(2)}(x_3, x_4) \rangle = \delta(x_1 - x_3)\delta(x_2 - x_4) + \delta(x_1 - x_4)\delta(x_2 - x_3)$$

ME 639-Turbulence

G. Ahmadi

Wiener-Hermite Expansions Clarkson University

Wiener -Hermite Series

$$u(x) = \underbrace{\int K^{(1)}(x - x_1) H^{(1)}(x_1) dx_1}_{\text{Gaussian}} + \underbrace{\iint K^{(2)}(x - x_1, x - x_2) H^{(2)}(x_1, x_2) H^{(2)}(x_1, x_2) dx_1 dx_2}_{\text{Non-Gaussian}} + \underbrace{\iiint K^{(3)}(x - x_1, x - x_2, x - x_3) H^{(3)}(x_1, x_2, x_3) dx_1 dx_2 dx_3 + \dots}_{\text{Non-Gaussian}}$$

ME 639-Turbulence

G. Ahmadi

Winer-Hermite Model for the Burger Equation Clarkson University

Burger Equation

$$\frac{\partial u(x, t)}{\partial t} + u \frac{\partial u}{\partial x} = \nu \frac{\partial^2 u}{\partial x^2}$$

ME 639-Turbulence

G. Ahmadi

Wiener-Hermite Model for the Burger Equation

Clarkson
University

$$\left\{ \begin{array}{l} \left(\frac{\partial}{\partial t} - v \frac{\partial^2}{\partial x^2} \right) \mathbf{K}^{(1)}(x - x') \\ + 2 \frac{\partial}{\partial x} \int dx_1 \mathbf{K}^{(1)}(x - x_1) \mathbf{K}^{(2)}(x - x_1, x - x') = 0 \\ \left(\frac{\partial}{\partial t} - v \frac{\partial^2}{\partial x^2} \right) \mathbf{K}^{(2)}(x - x', x - x'') \\ + \frac{1}{2} \frac{\partial}{\partial x} \int [\mathbf{K}^{(1)}(x - x') \mathbf{K}^{(1)}(x - x'')] = 0 \end{array} \right.$$

ME 639-Turbulence

G. Ahmadi

Wiener-Hermite Expansions

Clarkson
University

Concluding Remarks

- Expansion of a function
- Orthogonal set
- Wiener-Hermite orthogonal set
- Expansion of a random function
- Burger Equation
- W-H expansion for the Burger Eq.

ME 639-Turbulence

G. Ahmadi

Wiener-Hermite Expansions

Clarkson
University

Thank you!

Questions?

ME 639-Turbulence

G. Ahmadi