ME 639 - Turbulence

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Wiener-Hermite Expansions Method

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Wiener-Hermite Expansions

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Outline

- Expansion of a function
- Orthogonal set
- Wiener-Hermite orthogonal set
- Expansion of a random function
- Burger Equation
- W-H expansion for the Burger Eq.

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Let $\varphi_n(x)$ be an orthogonal set

$$\int \varphi_n \varphi_m dx = \delta_{nm} \|\varphi_n\|$$

$$\|\varphi_n\| = \int \varphi_n^2 dx$$

Let u(x) be an arbitrary function

$$u(x) = \sum_n c_n \varphi_n(x)$$

$$c_n = \frac{\int u\varphi_n dx}{\|\varphi_n\|}$$

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Let a(x) be a white noise process

$$< a(x) >= 0$$

$$< a(x_1) a(x_2) >= \delta(x_1 - x_2)$$

Wiener -Hermite base

$$H^{(0)}(x) = 1$$

$$H^{(1)}(x) = a(x)$$

$$H^{(2)}(x_1, x_2) = a(x_1) a(x_2) - \delta(x_1 - x_2)$$

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Wiener -Hermite base

$$H^{(3)}(x_1,x_2,x_3)=a(x_1)a(x_2)a(x_3)-a(x_1)\delta(x_2-x_3)$$
$$-a(x_2)\delta(x_3-x_1)-a(x_3)\delta(x_1-x_2)$$

$$<H^{(i)}H^{(j)}>=0 \qquad i\neq j$$

Wiener -Hermite set is complete

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Wiener -Hermite base

$$<H^{(0)}(x)H^{(0)}(x)>=1$$

$$<H^{(1)}(x_1)H^{(1)}(x_2)>=\delta(x_1-x_2)$$

$$<H^{(2)}(x_1,x_2)H^{(2)}(x_3,x_3)>=\delta(x_1-x_3)\delta(x_2-x_4)$$
$$+\delta(x_1-x_4)(x_2-x_3)$$

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Wiener -Hermite Series

$$u(x)=\underbrace{\int K^{(1)}(x-x_1)H^{(1)}(x_1)dx_1}_{\text{Gaussian}}$$
$$+\underbrace{\int\int K^{(2)}(x-x_1,x-x_2)H^{(2)}(x_1,x_2)H^{(2)}(x_1,x_2)dx_1dx_2}_{\text{Non-Gaussian}}$$
$$+\underbrace{\int\int\int K^{3}(x-x_1,x-x_2,x-x_3)H^{(3)}(x_1,x_2,x_3)dx_1dx_2dx_3+\ldots}_{\text{Non-Gaussian}}$$

Winer-Hermite Model for the Burger Equation

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Burger Equation

$$\frac{\partial u(x,t)}{\partial t}+u\frac{\partial u}{\partial x}=\nu\frac{\partial^2 u}{\partial x^2}$$

Winer-Hermite Model for the Burger Equation

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$$\begin{cases} \left(\frac{\partial}{\partial t} - \nu \frac{\partial^2}{\partial x^2}\right) K^{(1)}(x - x') \\ + 2\frac{\partial}{\partial x}\int dx_1 K^{(1)}(x - x_1) K^{(2)}(x - x_1, x - x') = 0 \\ \qquad \left(\frac{\partial}{\partial t} - \nu \frac{\partial^2}{\partial x^2}\right) K^{(2)}(x - x', x - x'') \\ \qquad + \frac{1}{2}\frac{\partial}{\partial x}\int \left[K^{(1)}(x - x') K^{(1)}(x - x'')\right] = 0 \end{cases}$$

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Concluding Remarks

- Expansion of a function
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- Wiener-Hermite orthogonal set
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Thank you!

Questions?