ME 639 - Turbulence

Clarkson University

Large-Eddy Simulation

Goodarz Ahmadi

Department of Mechanical and Aeronautical Engineering
Clarkson University
Potsdam, NY 13699-5725

Large-Eddy Simulation

Clarkson University

Outline

- Filtering
- Large Scales and Subgrid Scales
- Subgrid Scales Stresses
- Leonard Stress
- Smogorinski Model
- Cross Stress

Large-Eddy Simulation

Clarkson University

Mass

$$\frac{\partial v_i}{\partial x_i} = 0$$

Momentum

$$\rho\left(\frac{\partial v_i}{\partial t} + v_j \frac{\partial v_i}{\partial x_j}\right) = \frac{\partial t_{ji}}{\partial x_j} + \rho f_i$$

Newtonian

$$t_{ij} = -p\delta_{ij} + \mu\left(v_{i,j} + v_{j,i}\right)$$

Large-Eddy Simulation

Clarkson University

Decomposition

Large Scale

Subgrid Scale

$$v_i = \overline{v}_i + v'_i \qquad t_{ij} = \overline{t}_{ij} + t'_{ij} \qquad p = \overline{p} + p'$$

Filter (Gaussian)

Filtered Large Scale

$$\overline{\phi}(\mathbf{x}) = \int_D G(\mathbf{x}, \mathbf{x}')\phi(\mathbf{x}')d\mathbf{x}'$$

Note That

$$\overline{\overline{\phi}} \neq \overline{\phi} \qquad \overline{\phi'} \neq 0$$

Filtered Equations

Clarkson University

$$\frac{\partial \overline{v_i}}{\partial x_i} = 0$$

$$\rho\left(\frac{\partial \overline{v_i}}{\partial t} + \overline{v_j}\frac{\partial \overline{v_i}}{\partial x_j}\right) = \frac{\partial \overline{t_{ji}}}{\partial x_j} + \frac{\partial t^S_{ji}}{\partial x_j} + \rho f_i$$

Subgrid-Scale Stress Tensor

$$t^S_{ji} = -\rho\left(\overline{v_j v_i} - \overline{v_j}\,\overline{v_i}\right)$$

Subgrid-Scale Stress Tensor

Clarkson University

$$t^S_{ji} = t^R_{ji} + t^L_{ji} + t^C_{ji}$$

Reynolds Stresses → $t^R_{ji} = -\rho\overline{v'_j v'_i}$

Leonard Stresses → $t^L_{ji} = -\rho\left(\overline{\overline{v_j}\,\overline{v_i}} - \overline{v_j}\,\overline{v_i}\right)$

Cross Stresses → $t^C_{ji} = -\rho\left(\overline{\overline{v_j}\,v'_i} + \overline{\overline{v_i}\,v'_j}\right)$

Reynolds Stresses

Clarkson University

$$t^R_{ji} = 2\nu_T \overline{D_{ji}}$$

$$\overline{D_{ji}} = \frac{1}{2}\left(\frac{\partial \overline{v_j}}{\partial x_i} + \frac{\partial \overline{v_i}}{\partial x_j}\right)$$

Smagorinsky Model → $\nu_T = (c_S\Delta)^2\left(\overline{D_{k\ell}}\,\overline{D_{k\ell}}\right)^{\frac{1}{2}}$

Grid Size

$$\Delta = (\Delta_1\Delta_2\Delta_3)^{\frac{1}{3}}$$

Smagorinsky Constant

$$c_S \approx 0.21$$

Cross Stresses

Clarkson University

$$t^c_{ji} = -\rho c_r\left(\overline{\overline{v_j}\,\overline{v_i}} - \overline{\overline{v_j}}\,\overline{\overline{v_i}}\right)$$

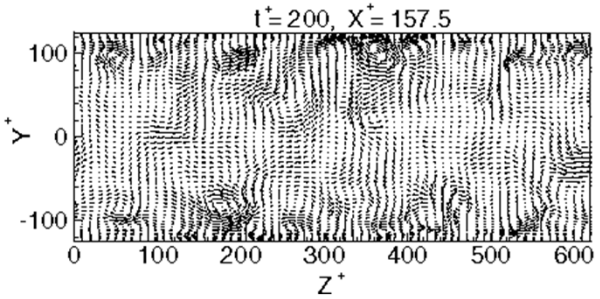
Speziale Galilean Invariance → $c_r = 1$

Leonard Stresses are Evaluated

→ $t^L_{ji} = -\rho\left(\overline{\overline{v_j}\,\overline{v_i}} - \overline{v_j}\,\overline{v_i}\right)$

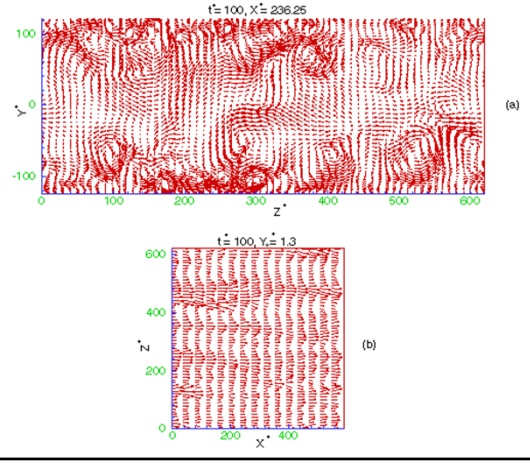
Turbulent Channel Flow-DNS

Clarkson University



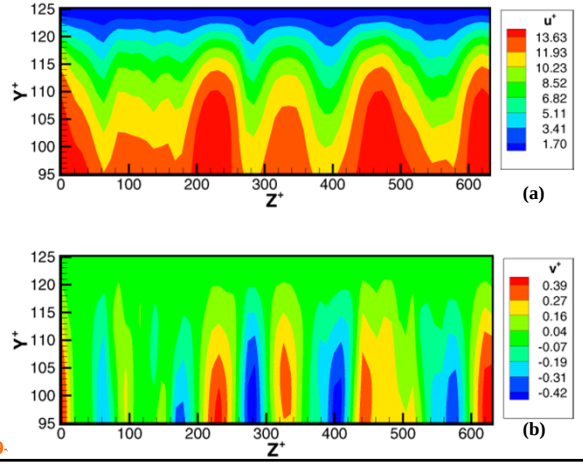
Turbulent Channel Flow-DNS

Clarkson University



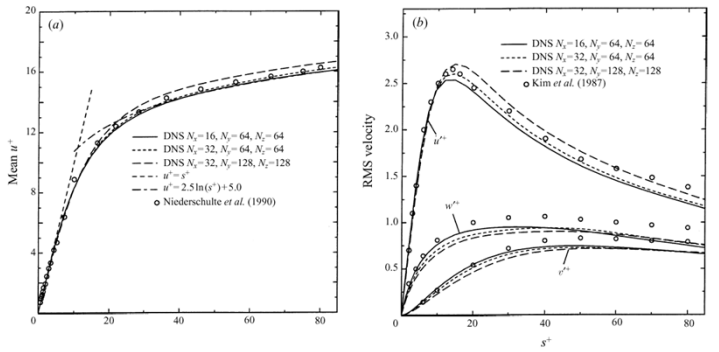
Turbulent Channel Flow-DNS

Clarkson University



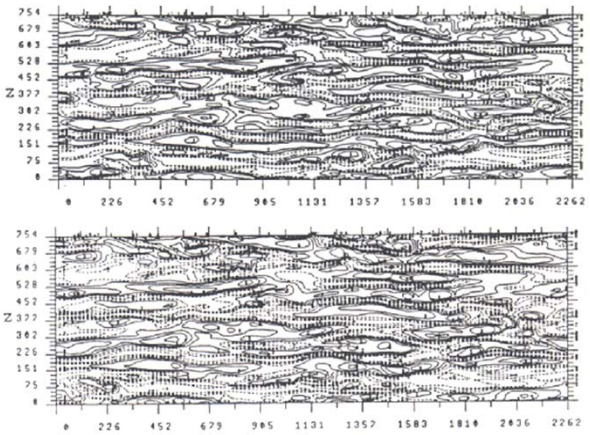
DNS

Clarkson University



LES

Clarkson University



Conclusions

Clarkson University

- LES captures computes for the large eddies of turbulence while the sub-grid scales are modeled.
- LES can provide more accurate description of turbulent flows.
- Smagorinski model provide a reasonable approximation.

Large-Eddy Simulation

Clarkson University

Thank you!

Questions?