ME 639 - Turbulence

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Eddy Viscosity Models

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Eddy Viscosity Models

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Outline

- Turbulent Boundary Layer Flows
- Eddy Viscosity Models
- Simple Shear Flows

Eddy Viscosity Models - Boundary Layer over a Flat Plate

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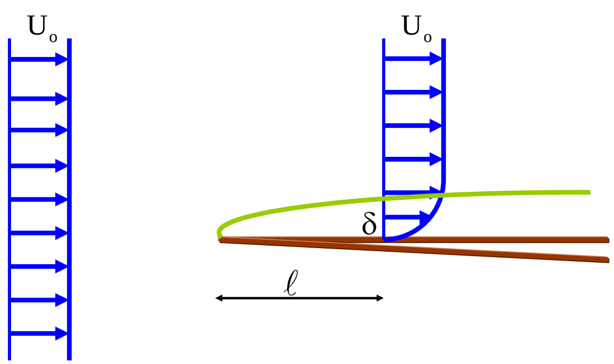

Inertial Sublayer

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Eddy Viscosity

$$\nu_T = ku^* y$$

$$\frac{\tau_0}{\rho} = u^{*2} = \nu_T \frac{dU}{dy} = ku^* y \frac{dU}{dy}$$

$$\frac{dU}{dy} = \frac{1}{k}\frac{u^*}{y}$$

Velocity Profile

$$\frac{U}{u^*} = \frac{1}{k}\ln y^+ + c$$

$$y^+ = \frac{yu^*}{\nu}$$

Reichardt Model

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Eddy Viscosity

$$\frac{\nu_T}{\nu} = k\left(y^+ - \delta_\ell^+ \tanh\frac{y^+}{\delta_\ell^+}\right)$$

$$y^+ \to 0 \qquad \frac{\nu_T}{\nu} \to y^{+3}$$

$\delta_\ell^+ = 12 \qquad k = 0.4$

$$\frac{\tau_{21}}{\rho} = (\nu + \nu_T)\frac{\partial U}{\partial y} \approx u^{*2}$$

Velocity Profile

$$U^+ = \frac{1}{k}\ln(1 + ky^+) + c\left[1 - e^{-\frac{y^+}{\delta_\ell^+}} - \frac{y^+}{\delta_\ell^+}e^{-0.33y^+}\right]$$

$c = 7.4$

Deissler Model

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Eddy Viscosity

$$\frac{\nu_T}{\nu} = aU^+y^+\left[1 - \exp(-aU^+y^+)\right]$$

$$y^+ \to 0 \qquad \frac{\nu_T}{\nu} \to y^{+4}$$

Rotta Model

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Eddy Viscosity

$$\tau_{21}^T = \ell\left[\nu + \ell_m^2\left|\frac{\partial U}{\partial y}\right|\right]\frac{\partial U}{\partial y} = \tau_0$$

$$\ell_m = \kappa(y - \delta_\ell)$$

Velocity Profile

$$U^+ = \frac{1}{2\kappa\ell_m^+}\left(1 - \sqrt{1 + 4\ell_m^{+2}}\right) + \frac{1}{\kappa}\ln\left(2\ell_m^+ + \sqrt{1 + 4\ell_m^{+2}}\right) + \delta_\ell^+$$

$$\ell_m^+ = \frac{u^*\ell_m}{\nu} \qquad \delta_\ell^+ = \frac{u^*\delta_\ell}{\nu}$$

Van Driest Model

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Eddy Viscosity

$$\upsilon_T = k^2y^2\left[1 - \exp\left(-\frac{y^+}{A}\right)\right]^2\frac{\partial U}{\partial y}$$

$k = 0.4 \qquad A = 27$

$y^+ \to 0 \qquad \upsilon_T \to y^{+4}$

$$\ell = ky\left[1 - e^{-\frac{y^+}{A}}\right]$$

Velocity Profile

$$U^+ = \frac{U}{u^*} = 2\int_0^{y^+}\frac{dy^+}{1 + \left\{1 + 4k^2y^{+2}\left[1 - \exp\left(-\frac{y^+}{A}\right)\right]^2\right\}}$$

Rannie Model

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Eddy Viscosity

$$\frac{\nu_T}{\nu} = \sinh^2 k_1 y^+$$

$$k_1 = 0.0688$$

Velocity Profile

$$U = \frac{1}{k_1}\tanh\left(k_1 y^+\right)$$

$$y^+ = 27.5$$

Eddy Viscosity Models

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Concluding Remarks

- Turbulent Boundary Layer Flows
- Eddy Viscosity Models
- Simple Shear Flows

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Thank you!

Questions?