Eddy Viscosity Models

Different eddy viscosity models for the wall region covering the viscous sublayer, the buffer zone and the inertial sublayer are described in this section. Near a wall,

$$\nu_T = ku^* y$$

leads to

$$\frac{\tau_0}{\rho} = u^{*2} = \nu_T \frac{dU}{dy} = ku^* y \frac{dU}{dy}$$

$$\frac{dU}{dy} = \frac{1}{k}\frac{u^*}{y}$$

$$\frac{U}{u^*} = \frac{1}{k}\ln y^+ + c \qquad \text{(valid in the inertial layer)}$$

here

$$y^+ = \frac{yu^*}{\nu}$$

Reichardt Model

Reichardt assumed

$$\frac{\nu_T}{\nu} = k\left(y^+ - \delta_\ell^+ \tanh\frac{y^+}{\delta_\ell^+}\right)$$

where d_ℓ^+ is the (laminar) viscous sublayer thickness. Note that

$$\text{as} \quad y^+ \to 0 \qquad \frac{\nu_T}{\nu} \to y^{+3}$$

Also

$$\frac{\tau_{21}}{\rho} = (\nu + \nu_T)\frac{\partial U}{\partial y} \approx u^{*2}$$

The velocity profile then approximately is given as

$$U^+ = \frac{1}{k}\ln\left(1 + ky^+\right) + c\left[1 - e^{-\frac{y^+}{\delta_\ell^+}} - \frac{y^+}{\delta_\ell^+} e^{-0.33y^+}\right]$$

where $k = 0.4$, $c = 7.4$, $\delta_\ell^+ = 12$

Deissler Model

Deissler assumed

$$\frac{\nu_T}{\nu} = aU^+ y^+ \left[1 - \exp\left(-aU^+ y^+\right)\right]$$

Here as $y^+ \to 0$, $\frac{\nu_T}{\nu} \to {y^+}^4$

Rotta Model

Rotta assumed

$$\tau_{21}^T = \ell\left[\nu + \ell_m^2 \left|\frac{\partial U}{\partial y}\right|\right]\frac{\partial U}{\partial y} = \tau_0$$

with

$$\ell_m = \kappa(y - \delta_\ell),$$

where δ_ℓ is the viscous sublayer length. Then

$$U^+ = \frac{1}{2\kappa\ell_m^+}\left(1 - \sqrt{1 + 4\ell_m^{+2}}\right) + \frac{1}{\kappa}\ln\left(2\ell_m^+ + \sqrt{1 + 4\ell_m^{+2}}\right) + \delta_\ell^+$$

where

$$\ell_m^+ = \frac{u^*\ell_m}{\nu}, \quad \delta_\ell^+ = \frac{u^*\delta_\ell}{\nu}$$

Van Driest Model

Modified mixing length:

$$\ell = ky\left[1 - e^{-\frac{y^+}{A}}\right]$$

$$\upsilon_T = k^2 y^2 \left[1 - \exp\left(-\frac{y^+}{A}\right)\right]^2 \frac{\partial U}{\partial y}$$

Here

and

$$k = 0.4 \qquad A = 27$$

$$y^+ \to 0 \quad \upsilon_T \to {y^+}^4$$

The corresponding velocity profile is given by

$$U^+ = \frac{U}{u^*} = 2\int_0^{y^+} \frac{dy^+}{1 + \left\{1 + 4k^2 {y^+}^2 \left[1 - \exp\left(-\frac{y^+}{A}\right)\right]^2\right\}}$$

Spalding Model

$$U^+ = \frac{U}{u^*}$$

$$y^+ = U^+ + c\left[e^{kU^+} - 1 - kU^+ - \frac{(kU^+)^2}{2!} - \frac{(kU^+)^3}{3!} + \frac{(kU^+)^4}{4!}\right]$$

$$c = e^{-kB}, \qquad B = 5.5, \qquad k = 0.4, \qquad c = 0.1108$$

Rannie Model

For viscous sublayer and buffer region, Rannie suggested

$$\frac{\nu_T}{\nu} = \sinh^2 k_1 y^+, \qquad k_1 = 0.0688$$

The corresponding velocity profile then becomes

$$U^+ = \frac{1}{k_1}\tanh\left(k_1 y^+\right)$$

It joints the logarithmic distribution at $y^+ = 27.5$.

Zero-Equation Model of Cebeci and Smith (1974)

Inner region $0 \le y \le y_c$:

$$\nu_T = \ell^2 \left|\frac{\partial U}{\partial y}\right| \gamma_{tr}, \qquad \ell = \kappa y\left[1 - \exp\left(-\frac{y}{A}\right)\right], \qquad (0 < y \le y_c)$$

Outer region $y_c < y \le \delta$:

$$\nu_T = \alpha \left| \int_0^\infty (U_0 - U)\,dy \right| \gamma_{tr} = 0.0168 U_0 \delta^* \qquad \text{for a wall boundary layer } R_\theta > 5000$$

where γ_{tr} is the intermitting factor.

The formulation is based on van Driest Approach in inner region

$$A = A^+ \frac{\nu}{Nu^*}$$

$$u^* = \left(\frac{\tau_0}{\rho}\right)^{\frac{1}{2}}, \quad N = \left\{\frac{P^+}{V_w^+}\left[1 - e^{11.8 V_w^+}\right] + e^{11.8 V_w^+}\right\}$$

$$P^+ = -\frac{\nu U_0}{u^{*3}} \frac{dU_0}{dx}, \qquad V^+ = \frac{V_w}{u^*}, \qquad A^{+|} = 26, \qquad V_w = V|_{wall}$$

For no mass transfer,

$$N = \left(1 - 11.8 P^+\right)^{\frac{1}{2}}$$

For $R_\theta < 5000$,

$$\alpha = 0.0168 \frac{1.55}{1 + \Pi}, \qquad \Pi = 0.55\left[1 - \exp\left(-0.243 z_1^{\frac{1}{2}} - 0.298 z_1\right)\right]$$

$$z_1 = \left(\frac{R_\theta}{425} - 1\right)$$

The intermitting actor for laminar-turbulent transition is given by

$$\gamma_{tr} = 1 - \exp\left\{ -G x_{tr} (x - x_{tr}) \int_{x_{t1}}^{x} \frac{1}{U_0} dx \right\}$$

$$G = \frac{1}{1200} \frac{U_0^3}{\nu^2} R_{x_{tr}}^{-1.34}, \quad R_{x_{tr}} = \frac{U_0 x_{tr}}{\nu},$$

where x_{tr} is the location of the onset of transition