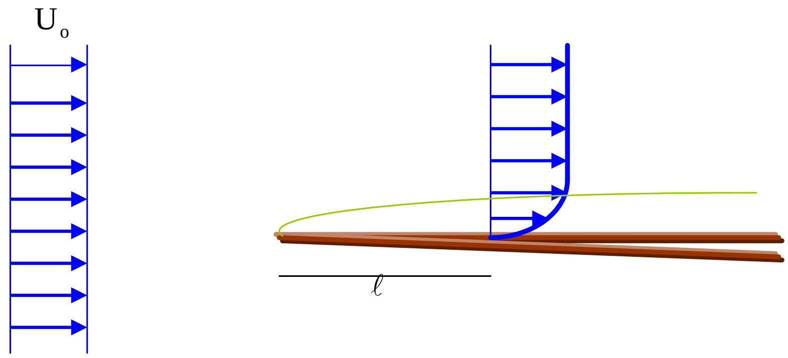
Flat Plate (No Pressure Gradient)

Boundary layer properties for laminar and turbulent flows are summarized in this section,



Laminar Boundary Layer

$$\frac{\delta}{x} = 4.96\,\mathrm{Re}_x^{-\frac{1}{2}}, \qquad C_f = 1.328\,\mathrm{Re}_\ell^{-\frac{1}{2}}, \qquad \mathrm{Re}_\ell = \frac{U_0 \ell}{\nu}$$

$$\text{Transition } \mathrm{Re}_{crit} \approx \begin{cases} 3.2\times10^5 \sim 10^6 \\ 5\times10^5 \sim 3\times10^6 \end{cases}$$

Turbulent Boundary Layer

Smooth Plate

$$\frac{\delta}{x} = 0.37\,\mathrm{Re}_x^{-\frac{1}{5}}, \qquad C_f' = 0.0577\,\mathrm{Re}_x^{-\frac{1}{5}} \qquad 5\times10^5 < \mathrm{Re}_x < 10^7$$

$$C_f = \frac{0.074}{\mathrm{Re}_\ell^{\frac{1}{5}}} - \frac{A}{\mathrm{Re}_\ell} \qquad 5\times10^5 < \mathrm{Re}_\ell < 10^7$$

Values of parameter A for smooth plates

R_{crit}	3×10^5	5×10^5	10^6	3×10^6
A	1050	1700	3300	8700

Prandtl-Schlichting (Drag Coefficient for Smooth Plates)

$$C_f = \frac{0.455}{(\log \mathrm{Re}_\ell)^{2.58}} - \frac{A}{\mathrm{Re}_\ell} \qquad \mathrm{Re}_\ell > 10^7$$

Schlichting Formula (Drag Coefficient)

Rough: $$C_f = \left(1.89 + 1.58 \log \frac{\ell}{e}\right)^{-2.5}$$ Rough Zone

Smooth: $$C_f = \frac{0.031}{\mathrm{Re}_\ell^{\frac{1}{7}}} - \frac{A}{\mathrm{Re}_\ell}$$ Transition Zone

Admissible Roughness

$$e_{odm} = \ell \frac{100}{\mathrm{Re}_\ell}$$

Boundary Layer Thickness

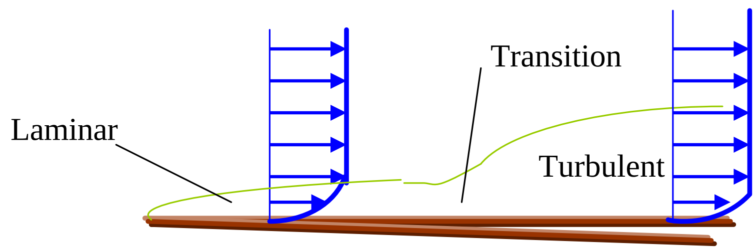
$$\frac{d}{x} = 0.16 \mathrm{Re}_x^{-\frac{1}{7}}$$

Turbulent Boundary Layer

Boundary Layer Transition

$$R_\delta\big|_{critical} \approx 1220 \;, \qquad R_\theta\big|_{critical} \approx 420$$

$$R_L\big|_{critical} = 5\times10^5\left(\sim 3\times10^6\right)$$



Boundary Layer Equations

$$U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial y} = -\frac{1}{\rho}\frac{dP_0}{dx} + \nu\frac{\partial^2 U}{\partial y^2} - \frac{\partial}{\partial y}\overline{u'v'}$$

$$P_0 = \overline{P} + \rho\overline{v'^2}$$

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0$$

Integral Equation (von Karman Momentum Integral)

$$\frac{d}{dx}\theta + \frac{2\theta+\delta^*}{U_0}\frac{dU_0}{dx} = \frac{\tau_0}{\rho U_0^2}$$

$$\delta^* = \int_0^\infty\left(1-\frac{U}{U_0}\right)dy = \text{Displacement Thickness}$$

$$\theta = \int_0^\infty\frac{U}{U_0}\left(1-\frac{U}{U_0}\right)dy = \text{Momentum Thickness}$$

$$H = \frac{\delta^*}{\theta} = \text{shape factor}$$

Power Law Velocity Approximation

$$\frac{U}{U_0}=\left(\frac{y}{\delta}\right)^{\frac{1}{n}}=\eta^{\frac{1}{n}}$$

$$\frac{U_{av}}{U_0}=\frac{n}{n+1}$$ where U_{av} = average velocity

$$\frac{U_0}{u^*}=c_1 R_\delta^{\frac{1}{n+1}}$$

$$\frac{c_f}{2}=c_1^{-2} R_\nu^{-\frac{2}{n+1}}$$

$$\frac{d^*}{d}=\frac{1}{n+1},\qquad \frac{\theta}{\delta}=\frac{n}{(n+1)(n+2)}$$

$$H=\frac{\delta^*}{\theta}=\frac{n+2}{n}$$

$$\frac{U_0\theta}{\nu}=\left(c_2\frac{U_0 x}{\nu}\right)^{\frac{n+1}{n+3}}$$

Note $\frac{1}{7}$ corresponds to the Blasius resistance law.

$$c_f \sim \left(\frac{U_0\delta}{\nu}\right)^{-\frac{1}{4}}.$$

$$\frac{U}{u^*}=8.3\left(\frac{u^* y}{\nu}\right)^{\frac{1}{7}}$$

$$\frac{U_0-U}{u^*}=0.6\left(1-\frac{y}{\delta}\right)^2 \qquad \text{for} \qquad \frac{y}{\delta}>0.15$$

Friction Coefficient

Ludwieg - Tillmann Expression

$$C_f = 0.246\times 10^{-0.678H}\left(\frac{U_0\theta}{\nu}\right)^{-0.268} \qquad 10^3 < R_q < 10^4, \qquad R_\theta = \frac{U_0\theta}{\nu}$$

Here H is a function $\frac{u^*}{U_0}$ or C_f.

In this range,

$$H \approx 1.36, \qquad C_f \approx 0.03 R_\theta^{-0.268}$$

which corresponds to a power-law velocity with $n = 6.45$.

$$R_\theta = 0.045 R_x^{0.79}, \qquad R_x = \frac{U_0 x}{\nu}$$

Experimental Data for Turbulent Boundary Layer
(Hinze, Turbulence, McGraw-Hill, 1975)