Turbulent Boundary Layer Flows

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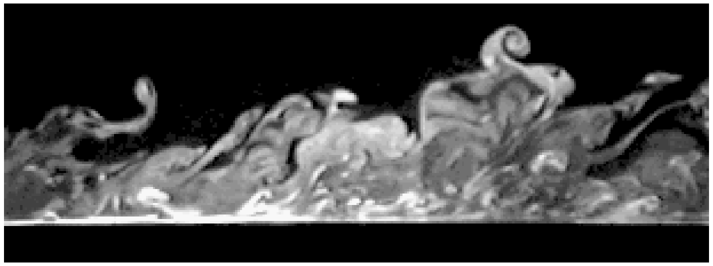
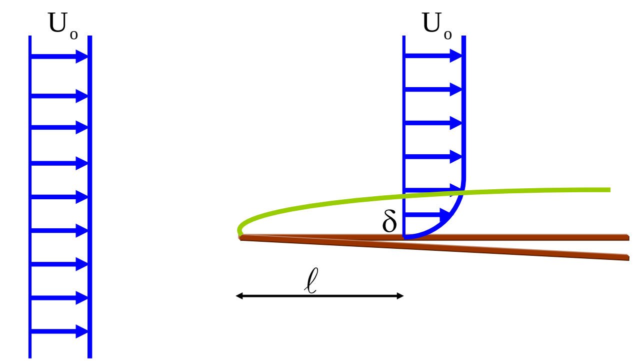


Objectives

- Turbulent Boundary Layer Flows
- Friction Laws
- Power Laws
- Momentum Integral
- Blasius Resistance



Laminar Boundary Layer

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Boundary Layer Thickness

$$\frac{\delta}{x} = 4.96\,\mathrm{Re}_x^{-\frac{1}{2}}$$

Friction Coefficient

$$C_f = 1.328\,\mathrm{Re}_\ell^{-\frac{1}{2}}$$

$$\mathrm{Re}_\ell = \frac{U_0 \ell}{\nu}$$

Critical Reynolds Number

$$\mathrm{Re}_{crit} \approx \begin{cases} 3.2\times10^5 \sim 10^6 \\ 5\times10^5 \sim 3\times10^6 \end{cases}$$

Turbulent Boundary Layer -Smooth

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Boundary Layer Thickness

$$\frac{\delta}{x} = 0.37\,\mathrm{Re}_x^{-\frac{1}{5}}$$

Friction Coefficient

$$C_f' = 0.0577\,\mathrm{Re}_x^{-\frac{1}{5}}$$

$$5\times10^5 < \mathrm{Re}_x < 10^7$$

Friction Coefficient including Transition

$$C_f = \frac{0.074}{\mathrm{Re}_\ell^{\frac{1}{5}}} - \frac{A}{\mathrm{Re}_\ell}$$

Turbulent Boundary Layer

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Prandtl-Schlichting

$$C_f = \frac{0.455}{(\log \mathrm{Re}_\ell)^{2.58}} - \frac{A}{\mathrm{Re}_\ell}$$

$$\mathrm{Re}_\ell > 10^7$$

Schlichting Rough Zone

$$C_f = \left(1.89 + 1.58 \log\frac{\ell}{e}\right)^{-2.5}$$

Boundary Layer Thickness

$$\frac{\delta}{x} = 0.16\,\mathrm{Re}_x^{-\frac{1}{7}}$$

Turbulent Boundary Layer

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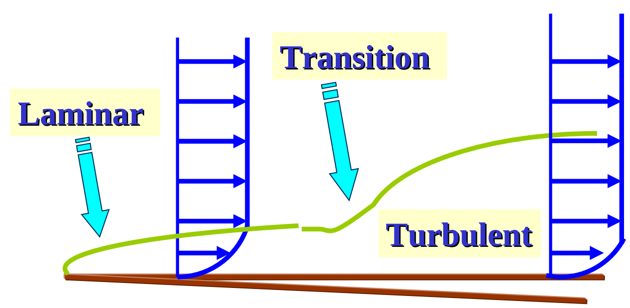

Boundary Layer Transition

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$$R_\delta \big|_{critical} \approx 1220$$

$$R_\theta \big|_{critical} \approx 420$$

$$R_L \big|_{critical} = 5\times10^5 \left(\sim 3\times10^6\right)$$

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Turbulent Boundary Layer Equations

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Momentum Equation

$$U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial y} = -\frac{1}{\rho}\frac{dP_0}{dx} + \nu\frac{\partial^2 U}{\partial y^2} - \frac{\partial}{\partial y}\overline{u'v'}$$

Pressure → $P_0 = \overline{P} + \rho\overline{v'^2}$

Mass → $\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0$

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Momentum Integral Method

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$$\frac{d}{dx}\theta + \frac{2\theta + \delta^*}{U_0}\frac{dU_0}{dx} = \frac{\tau_0}{\rho U_0^2}$$

Displacement Thickness → $\delta^* = \int_0^\infty \left(1 - \frac{U}{U_0}\right) dy$

Momentum Thickness → $\theta = \int_0^\infty \frac{U}{U_0}\left(1 - \frac{U}{U_0}\right) dy$

Shape Factor → $H = \frac{\delta^*}{\theta}$

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Power Law Velocities

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$$\frac{U}{U_0} = \left(\frac{y}{\delta}\right)^{\frac{1}{n}} = \eta^{\frac{1}{n}}$$

$$\frac{U_{av}}{U_0} = \frac{n}{n+1}$$

$$\frac{U_0}{u^*} = c_1 R_\delta^{\frac{1}{n+1}}$$

$$\frac{c_f}{2} = c_1^{-2} R_\nu^{-\frac{2}{n+1}}$$

$$\frac{\delta^*}{\delta} = \frac{1}{n+1}$$

$$\frac{\theta}{\delta} = \frac{n}{(n+1)(n+2)}$$

$$H = \frac{\delta^*}{\theta} = \frac{n+2}{n}$$

$$\frac{U_0\theta}{\nu} = \left(c_2\frac{U_0 x}{\nu}\right)^{\frac{n+1}{n+3}}$$

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Blasius Resistance Law

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$$c_f \sim \left(\frac{U_0 \delta}{\nu}\right)^{-\frac{1}{4}}$$

$$\frac{U}{u^*} = 8.3\left(\frac{u^* y}{\nu}\right)^{\frac{1}{7}}$$

$$\frac{U_0 - U}{u^*} = 0.6\left(1 - \frac{y}{\delta}\right)^2$$

$$\frac{y}{\delta} > 0.15$$

Turbulent Boundary Layer Friction Coefficient

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Ludwieg-Tillmann

$$C_f = 0.246 \times 10^{-0.678H}\left(\frac{U_0 \theta}{\nu}\right)^{-0.268}$$

$$10^3 < R_\theta < 10^4$$

$$H \approx 1.36$$

$$C_f \approx 0.03 R_\theta^{-0.268}$$

$$R_\theta = 0.045 R_x^{0.79}$$

$$n = 6.45$$

Turbulent Boundary Layer Flows

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Concluding Remarks

- Turbulent Boundary Layer Flows
- Friction Laws
- Power Laws
- Momentum Integral
- Blasius Resistance