ME 639 - Turbulence

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Turbulent Wake Flows

Goodarz Ahmadi

Department of Mechanical and Aeronautical Engineering
Clarkson University
Potsdam, NY 13699-5725

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Outline

- Turbulent Wake Flows
- Momentum Integral
- Similarity Variable
- Eddy Viscosity Model
- Similarity Solution

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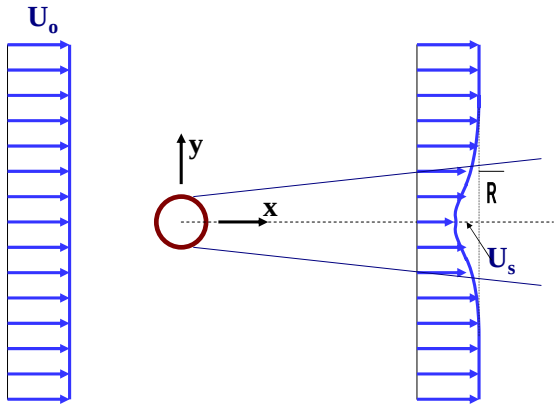

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Simplified Reynolds Equation

$$U_0 \frac{\partial U}{\partial x} + \frac{\partial}{\partial y}\overline{u'v'} = 0$$

Momentum Integral

$$\rho \int_{-\infty}^{+\infty} U_0 (U - U_0) dy = M = -\rho U_0^2 \theta$$

Momentum Thickness

$$\theta = \int_{-\infty}^{+\infty} \left(1 - \frac{U}{U_0}\right) dy$$

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Drag Coefficient

$$C_d = \frac{D}{\frac{1}{2}\rho U_0^2 d} = -\frac{M}{\frac{1}{2}\rho U_0^2 d} = \frac{\rho U_0^2 \theta}{\frac{1}{2}\rho U_0^2 d} = \frac{2\theta}{d}$$

$Re_d \sim 10^3 \text{ to } 3\times 10^5 \Rightarrow c_d = 1 \Rightarrow \theta \approx \frac{d}{2}$

Self Similar Solutions

$$\frac{U_0 - U}{U_s} = f\left(\frac{y}{\ell}\right) \qquad -\overline{u'v'} = U_s^2 g\left(\frac{y}{\ell}\right)$$

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Similarity Variables

$$U_s = Ax^m \qquad \ell = Bx^n$$

Then

$$\frac{\partial U}{\partial x} \sim \frac{U_s}{x} \sim x^{m-1} \qquad \frac{\partial \overline{u'v'}}{\partial y} \sim \frac{U_s^2}{\ell} \sim x^{2m-n}$$

Momentum and Momentum Integral

$$m - 1 = 2m - n \qquad m + n = 0$$

$$\{n - m = 1 \quad m = -n\} \Rightarrow n = \frac{1}{2} \quad m = -\frac{1}{2}$$

$$U_0 = Ax^{-\frac{1}{2}} \qquad \ell = Bx^{\frac{1}{2}}$$

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$$\xi = \frac{y}{\ell} = \frac{y}{Bx^{\frac{1}{2}}} \qquad \frac{\partial \xi}{\partial y} = \frac{1}{Bx^{\frac{1}{2}}} \qquad \frac{\partial \xi}{\partial x} = -\frac{1}{2}\frac{y}{Bx^{-\frac{3}{2}}} = -\frac{\xi}{2x}$$

Mean Velocity

$$U_0 - U = U_s f(\xi) = Ax^{-\frac{1}{2}} f(\xi)$$

Turbulence Shear Stress

$$\overline{u'v'} = -U_s^2 g(\xi) = -A^2 x^{-1} g(\xi)$$

Momentum Equation

$$\frac{U_0 B}{2A}(f + \xi f') = g'$$

Eddy Viscosity

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$$-\overline{u'v'} = \nu_T \frac{\partial U}{\partial y} \qquad \nu_T = -\frac{\overline{u'v'}}{\frac{\partial U}{\partial y}} = -\frac{-U_s^2 g}{-U_s f' \frac{1}{\ell}} = -U_s \ell \frac{g}{f'}$$

Turbulent Reynolds Number

$$\frac{\nu_T}{U_s \ell} = -\frac{g}{f'} = \frac{1}{R_T} \qquad R_T = \frac{U_s \ell}{\nu_T}$$

$\nu_T \approx \text{Cons tan t}$

$$g = -\frac{1}{R_T} f'$$

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$$\left(\xi f\right)' + f'' = 0$$

$$\xi f + f' = \text{const} = 0$$

Mean Velocity

$$\frac{U_0 - U}{U_s} = f = e^{-\frac{\xi^2}{2}}$$

Turbulence Shear Stress

$$g = -\frac{\xi}{R_T} e^{-\frac{\xi^2}{2}} = -\frac{\overline{u'v'}}{U_s^2}$$

$$\theta = \int_{-\infty}^{+\infty} \left(1 - \frac{U}{U_0}\right) dy = \frac{U_s}{U_0} \ell \sqrt{2\pi}$$

Momentum Thickness

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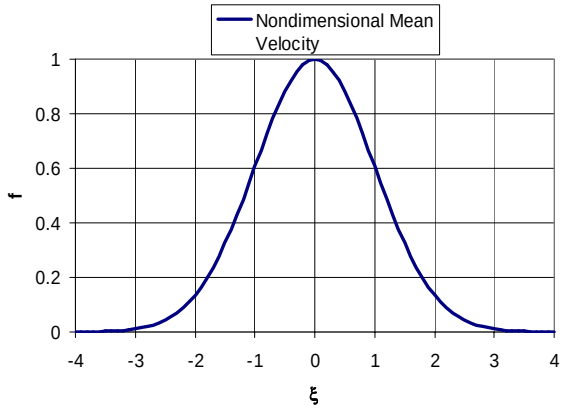

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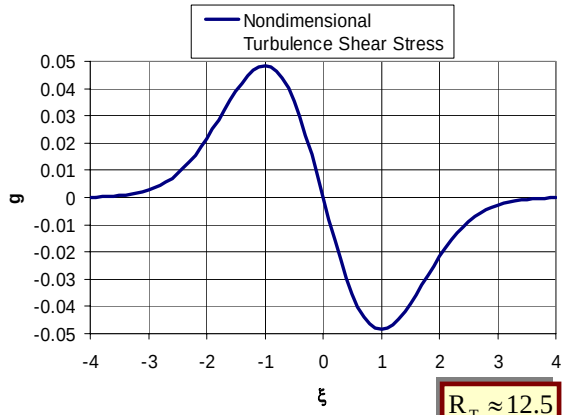

$R_T \approx 12.5$

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Similarity Variables

$$\begin{cases} \dfrac{U_s}{U_0} = \left(\dfrac{\theta}{x}\right)^{\frac{1}{2}} \left(\dfrac{R_T}{2\sqrt{2\pi}}\right)^{\frac{1}{2}} = 1.58 \left(\dfrac{\theta}{x}\right)^{\frac{1}{2}} \\ \dfrac{\ell}{\theta} = \left(\dfrac{x}{\theta}\right)^{\frac{1}{2}} \left(\dfrac{2}{\sqrt{2\pi} R_T}\right)^{\frac{1}{2}} = 0.252 \left(\dfrac{x}{\theta}\right)^{\frac{1}{2}} \end{cases}$$

$R_T \approx 12.5$

$$U_s \ell = \frac{\theta U_0}{\sqrt{2\pi}} = 0.4 \theta U_0$$

$$\nu_T = \frac{U_s \ell}{12.5} = 0.0319\, U_0 \theta$$