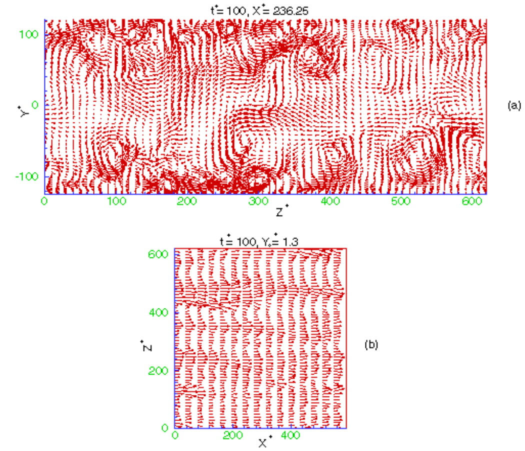
ME 639 - Turbulence

Turbulent Flow Between Two Parallel Plates

Goodarz Ahmadi

Department of Mechanical and Aeronautical Engineering
Clarkson University
Potsdam, NY 13699-5725

Turbulent Flow Between Two Parallel Plates

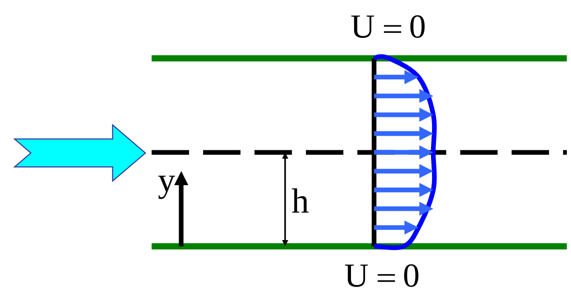


Turbulent Flow Between Two Parallel Plates

Outline

- Flows Between Two Parallel Plates
- Near Wall Flows
- Law of the Wall
- Velocity Defect Law
- Near Wall Scales

Turbulent Flow Between Two Parallel Plates



Turbulent Flow Between Two Parallel Plates

Clarkson University

Reynolds Equation

$$U_j \frac{\partial U_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial P}{\partial x_i} + \nu \frac{\partial^2 U_i}{\partial x_j \partial x_j} - \frac{\partial \overline{u_i' u_j'}}{\partial x_j}$$

Parallel Flows → $\mathbf{U} = (U(y),0,0)$

$$0 = -\frac{1}{\rho}\frac{\partial P}{\partial x} - \frac{d}{dy}\overline{u'v'} + \nu \frac{d^2 U}{dy^2}$$

$$0 = -\frac{1}{\rho}\frac{\partial P}{\partial y} - \frac{d}{dy}\overline{v'^2}$$

ME 639-Turbulence G. Ahmadi

Turbulent Duct Flows

Clarkson University

$$\frac{P}{\rho} + \overline{v'^2} = \frac{P_0}{\rho} \rightarrow \frac{\partial P}{\partial x} = \frac{dP_0}{dx}$$

Momentum Equation

$$0 = -\frac{1}{\rho}\frac{dP_0}{dx} - \frac{d}{dy}\overline{u'v'} + \nu \frac{d^2 U}{dy^2}$$

Shear Velocity

$$\nu \left.\frac{dU}{dy}\right|_{y=0} = \frac{\tau_0}{\rho} = u^{*2}$$

Momentum Equation

$$\frac{y}{\rho}\frac{dP_0}{dx} - \overline{u'v'} + \nu \frac{dU}{dy} - u^{*2} = 0$$

ME 639-Turbulence G. Ahmadi

Turbulent Duct Flows

Clarkson University

Momentum Equation at y=h

$$-\frac{h}{\rho}\frac{dP_0}{dx} = u^{*2}$$

Momentum Equation

$$-\overline{u'v'} + \nu \frac{dU}{dy} = u^{*2}\left(1 - \frac{y}{h}\right)$$

Core Region

$$\eta = \frac{y}{h}$$

$$U^+ = \frac{U}{u^*}$$

Nondimensional Momentum Equation

$$-\frac{\overline{u'v'}}{u^{*2}} + \frac{1}{R^*}\frac{d}{d\eta}(U^+) = 1 - \eta$$

Reynolds Number → $R^* = \frac{u^* h}{\nu}$

ME 639-Turbulence G. Ahmadi

Turbulent Duct Flows

Clarkson University

Near Wall Region

Nondimensional Momentum Equation

$$y^+ = \frac{yu^*}{\nu} \rightarrow -\frac{\overline{u'v'}}{u^{*2}} + \frac{dU^+}{dy^+} = 1 - \frac{1}{R^*}y^+$$

$R^* \to \infty$, $\eta \sim 1$ — **Core Region** → $-\frac{\overline{u'v'}}{u^{*2}} = 1 - \eta$

$R^* \to \infty$, $y^+ \sim 1$ — **Wall Region** → $-\frac{\overline{u'v'}}{u^{*2}} + \frac{dU^+}{dy^+} = 1$

ME 639-Turbulence G. Ahmadi

Turbulent Duct Flows

Law of the Wall

$$\begin{Bmatrix} U^+ = f(y^+) \\ -\dfrac{\overline{u'v'}}{u^{*2}} = g(y^+) \end{Bmatrix}$$

Velocity Defect Law

$$\frac{U - U_0}{u^*} = F(\eta)$$

Velocity Gradient

$$\frac{dU}{dy} = \frac{u^{*2}}{\nu}\frac{df}{dy^+}$$

$$\frac{dU}{dy} = \frac{u^*}{h}\frac{dF}{d\eta}$$

Inertial Sublayer

In the region that

$$\eta \to 0$$
$$y^+ \to \infty$$

$$\frac{dU}{dy} = \frac{u^{*2}}{\nu}\frac{df}{dy^+} = \frac{u^*}{h}\frac{dF}{d\eta}$$

$$y^+\frac{df(y^+)}{dy^+} = \eta\frac{dF(\eta)}{d\eta} = \frac{1}{\kappa} = \text{const.}$$

$$\eta << 1$$

$$F(\eta) = \frac{1}{\kappa}\ln\eta + c_1$$

$$y^+ >> 1$$

$$f(y^+) = \frac{1}{\kappa}\ln y^+ + c_2$$

Inertial Sublayer

In Inertial Sublayer

$$-\frac{\overline{u'v'}}{u^{*2}} = 1$$

Velocity Defect Law

$$\frac{U - U_0}{u^*} = \frac{1}{\kappa}\ln\eta + c_1$$

Law of the Wall

$$\frac{U}{u^*} = \frac{1}{\kappa}\ln y^+ + c_2$$

$$\frac{U_0}{u^*} = \frac{1}{\kappa}\ln R^* + c_2 - c_1$$

$$R^* = \frac{u^* h}{\nu}$$

Logarithmic Friction Law

Turbulent Pipe Flows

$$\eta = \frac{y}{r_0}$$

$$R^* = \frac{u^* r_0}{\nu}$$

$$\kappa = 0.4$$

Law of the Wall

$$U^+ = \frac{U}{U^*} = 2.5\ln y^+ + 5$$

Velocity Defect Law

$$\frac{U - U_0}{u^*} = 2.5\ln\eta - 1$$

Logarithmic Friction Law

$$\frac{U_0}{u^*} = 2.5R^* + 6$$

Wall Layer

Clarkson University

Law of the Wake

$$W(\eta) = 1 - 2.5 \ln \eta + F(\eta)$$

$$W(\eta) = \frac{1}{2}\left[\sin \pi(\eta - \frac{1}{2}) + 1\right]$$

Viscous Sublayer

$$\frac{dU^+}{dy^+} = 1$$

$$U^+ = y^+$$

Inertial Sublayer

Clarkson University

$$-\overline{u'v'} \approx u^{*2}$$

$$\frac{\partial U}{\partial y} \approx \frac{u^*}{\kappa y}$$

$$\text{Production} = -\overline{u'v'}\frac{\partial U}{\partial y} = \frac{u^{*3}}{\kappa y}$$

$$\varepsilon = \frac{u^{*3}}{\kappa y}$$

Kolmogorov Length Scale

$$\eta = \left(\frac{\nu^3}{\varepsilon}\right)^{\frac{1}{4}}$$

$$\eta^+ = \frac{\eta u^*}{\nu}$$

$$\eta^+ = (\kappa y^+)^{\frac{1}{4}}$$

Inertial Sublayer

Clarkson University

Turbulence Macroscale

$$\Lambda = \kappa y$$

$$\Lambda^+ = \frac{\Lambda u^*}{\nu} = \kappa y^+$$

y^+	$\eta^+ = (\kappa y^+)^{\frac{1}{4}}$	$\Lambda^+ = \kappa y^+$
5	1.2	2
12	1.5	4
40	2	16
200	3	80
1000	4.5	400

Turbulence Scales

Clarkson University