ME 639 - Turbulence

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Correlation, Spectrum, and Scales

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Correlation, Spectrum, and Scales

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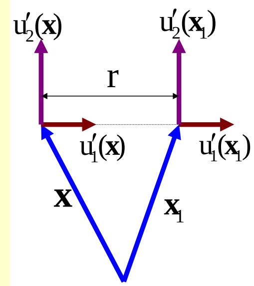
Outline

- Two-Point Correlation Tensor
- Longitudinal and Lateral Integral Scales
- Taylor Microscales
- Energy Spectrum
- Relations between Scales
- Order of Magnitude Analysis



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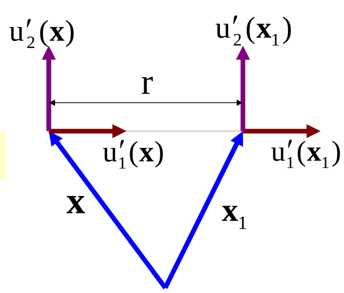
Two-Point Correlation Tensor

$$Q_{ij}(\mathbf{x},\mathbf{x_1}) = \overline{u'_i(\mathbf{x})u'_j(\mathbf{x_1})}$$

Homogenous Turbulence

$$Q_{ij}(\mathbf{x},\mathbf{x_1}) = Q_{ij}(\mathbf{r})$$

$$\mathbf{r} = \mathbf{x_1} - \mathbf{x_2}$$



Longitudinal Correlation Coefficient

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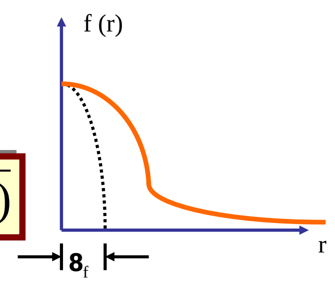
$$f(r) = \frac{Q_{11}}{u_1^2}$$

$$Q_{11} = \overline{u'_1(\mathbf{x})u'_1(\mathbf{x_1})}$$

$$u_1^2 = \overline{u_1'^2(\mathbf{x})} = \overline{u_1'^2(\mathbf{x_1})}$$

$$f(r) = f(-r)$$



Lateral Correlation Coefficient

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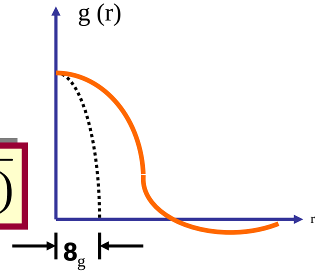
$$g(r)=\frac{Q_{22}}{u_2^2}$$

$$Q_{22}=\overline{u_2'(\mathbf{x})u_2'(\mathbf{x}_1)}$$

$$g(r)=g(-r)$$



Taylor's Microscales

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Longitudinal Microscale

$$\lambda_f^2=-\frac{2}{f''(0)}$$

Lateral Microscale

$$\lambda_g^2=-\frac{2}{g''(0)}$$

$$g(r)=1+\frac{1}{2!}r^2g''(0)+\cdots\approx 1-\frac{r^2}{\lambda_g^2}$$

Integral Scales, Macroscales

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Longitudinal Macroscale →

$$\Lambda_f=\int_0^\infty f(r)dr$$

Lateral Macroscale →

$$\Lambda_g=\int_0^\infty g(r)dr$$

Eulerian Time Correlation

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$$R_E(\tau)=\frac{\overline{u_1'(\mathbf{x},t)u_1'(\mathbf{x},t+\tau)}}{u_1^2}$$

Microscale

$$\tau_E^2=-\frac{2}{R_E''(0)}$$

Macroscale

$$T_E=\int_0^\infty R_E(\tau)d\tau$$

Frozen Field Approximation

$$\Lambda_f\approx UT_E$$

$$\lambda_f\approx U\tau_E$$

$$\frac{\partial}{\partial t}=-U\frac{\partial}{\partial x}$$

$$f(U\tau)\approx R_E(\tau)$$

Lagrangian Time Correlation

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$$R_L(\tau) = \frac{\overline{v'_L(t)v'_L(t+\tau)}}{\overline{v'^2_L}}$$

Lagrangian Time Microscale

$$\tau_L^2 = -\frac{2}{R''_L(0)}$$

Lagrangian Time Macroscale

$$T_L = \int_0^\infty R_L(\tau)\,d\tau$$

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Energy Spectrum

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$$E_{ij}(\mathbf{k}) = \frac{1}{8\pi^3}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} Q_{ij}(\mathbf{x})e^{-i\mathbf{k}\cdot\mathbf{x}}\,\mathbf{dx}$$

$$Q_{ij}(\mathbf{x}) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} E_{ij}(\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{x}}\,\mathbf{dk}$$

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One-D Energy Spectrum

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$$E_1(k_1) = \frac{u_1^2}{\pi}\int_{-\infty}^{+\infty} f(x_1)e^{-ik_1x_1}\,dx_1$$

$$u_1^2 f(x_1) = \frac{1}{2}\int_{-\infty}^{+\infty} E_1(k_1)e^{ik_1x_1}\,dk_1$$

$$E_1(k_1) = \frac{2u_1^2}{\pi}\int_0^\infty f(x_1)\cos k_1x_1\,dx_1$$

$$u_1^2 f(x_1) = \int_0^\infty E_1(k_1)\cos k_1x_1\,dk_1$$

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Sample Correlation and Spectrum

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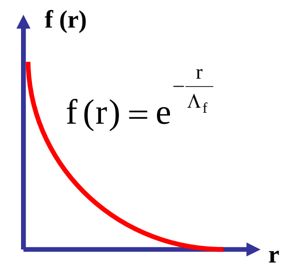

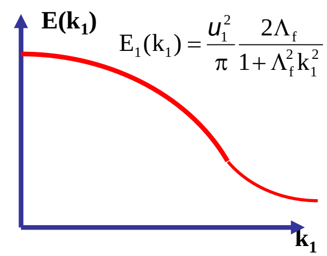

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Estimates for the Taylor Microscales

Energy Dissipation

$$\varepsilon = \nu \overline{\frac{\partial u'_i}{\partial x_j}\frac{\partial u'_i}{\partial x_j}}$$

Isotropic Turbulence

$$\varepsilon = 15\nu \overline{\left(\frac{\partial u'_1}{\partial x_1}\right)^2} = -15\nu \overline{u'^2_1} f''(0)$$

$$\varepsilon = 30\nu \frac{\overline{u'^2_1}}{\lambda_f^2} = 30\nu \frac{u^2}{\lambda_f^2} = 15\nu \frac{u^2}{\lambda_g^2}$$

$$\lambda_f = \lambda_g \sqrt{2}$$

Estimates for the Taylor Microscales

Macroscopic Estimate

$$\varepsilon = A\frac{u^3}{\Lambda} = 30\nu \frac{u^2}{\lambda_f^2}$$

$$\frac{\lambda_f}{\Lambda} = \sqrt{\frac{30}{A}} R_\Lambda^{-1/2} \qquad \frac{\lambda_f}{\Lambda} << 1 \qquad R_\Lambda = \frac{u\Lambda}{\nu} >> 1$$

$$\frac{\lambda_g}{\Lambda} = \sqrt{\frac{15}{A}} R_\Lambda^{-1/2} \qquad \frac{\lambda_g}{\Lambda} = \frac{15}{A} R_\lambda^{-1} \qquad R_\lambda = \frac{u\lambda}{\nu}$$

Relationships between the Scales

Taylor-Kolmogorov

$$\frac{\lambda_g}{\eta} = \left(\frac{225}{A}\right)^{1/4} R_\Lambda^{1/4} = 15^{1/4} R_\lambda^{1/2}$$

Kolmogorov-Time Scale

$$\frac{u}{\lambda_g} = 0.26\sqrt{\frac{\varepsilon}{\nu}} = \frac{0.26}{\tau} \qquad \tau = \frac{\eta}{\upsilon} = \sqrt{\frac{\nu}{\varepsilon}}$$

Relationships between the Scales

Velocity Gradient

$$\overline{\frac{\partial u'_i}{\partial x_j}\frac{\partial u'_i}{\partial x_j}} = \frac{\varepsilon}{\nu} \sim \left(\frac{u}{\lambda}\right)^2$$

Deformation Rate Tenor

$$\overline{d'_{ij}d'_{ij}} \sim \overline{\frac{\partial u'_i}{\partial x_j}\frac{\partial u'_i}{\partial x_j}} \sim \left(\frac{u}{\lambda}\right)^2$$

Viscosity

$$\frac{u^3}{\Lambda} \sim \nu \frac{u^2}{\lambda^2} \quad \Rightarrow \quad \nu \sim \frac{u\lambda^2}{\Lambda}$$

Relationships between the Scales

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Taylor/Integral

$$\frac{\lambda}{\Lambda} \sim R_{\Lambda}^{-1/2} \sim R_{\lambda}^{-1}$$

Kolmogorov/Integral

$$\frac{\eta}{\Lambda} \sim R_{\Lambda}^{-3/4} \sim R_{\lambda}^{-3/2}$$

Kolmogorov/Taylor

$$\frac{\eta}{\lambda} \sim R_{\Lambda}^{-1/4} \sim R_{\lambda}^{-1/2}$$

Different Scales

$$\eta^2 \Lambda = \lambda^3$$

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Concluding Remarks

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Thank you!

Questions?

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