ME 639 - Turbulence

Turbulence Energy Equation

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Energy Equation

Outline

- Mean Flow Energy Budget
- Turbulence Energy Budget
- Simple Shear Flows

Momentum Balance

Reynolds Equation

$$\rho\left(\frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial x_j}\right) = -\frac{\partial P}{\partial x_i} + \mu \frac{\partial^2 U_i}{\partial x_j \partial x_j} - \rho \frac{\partial \overline{u_i' u_j'}}{\partial x_j}$$

$$\frac{\partial U_i}{\partial x_i} = 0$$

Mean Energy Equation

$$\underbrace{\left[\frac{\partial}{\partial t} + U_j \frac{\partial}{\partial x_j}\right]\frac{U_i U_i}{2}}_{\text{convection}\ (u^3/\Lambda)} = \underbrace{-\frac{\partial}{\partial x_i}\left[U_i \frac{P}{\rho} + U_j \overline{u_i' u_j'}\right]}_{\text{diffusion by turbulence}\ (u^3/\Lambda)}$$

$$\underbrace{+\nu \frac{\partial^2}{\partial x_j \partial x_j}\frac{U_i U_i}{2}}_{\text{viscous diffusion}\ (\nu u^2/\Lambda^2)} \underbrace{- \nu \frac{\partial U_i}{\partial x_j}\frac{\partial U_i}{\partial x_j}}_{\text{viscous dissipation}\ (\nu u^2/\Lambda^2)} + \underbrace{\overline{u_i' u_j'}\frac{\partial U_i}{\partial x_j}}_{\text{fluctuation energy production}\ (u^3/\Lambda)}$$

Turbulence Energy Balance

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Navier-Stokes Eq. – Reynolds Equation⇒

Fluctuation Velocity Equation

$$\frac{\partial u_i'}{\partial t} + U_j \frac{\partial u_i'}{\partial x_j} + u_j' \frac{\partial U_i}{\partial x_j} + u_j' \frac{\partial u_i'}{\partial x_j} = -\frac{1}{\rho}\frac{\partial p'}{\partial x_j} + \nu \frac{\partial^2 u_i'}{\partial x_j \partial x_j} + \frac{\partial \overline{u_i' u_j'}}{\partial x_j}$$

Turbulence Kinetic Energy Balance

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$$\underbrace{\frac{\partial k}{\partial t} + U_j \frac{\partial k}{\partial x_j}}_{\text{Convection}} = \underbrace{-\overline{u_i' u_j'}\frac{\partial U_i}{\partial x_j}}_{\text{Production}} \underbrace{- \frac{\partial}{\partial x_j}\left[\frac{\overline{u_j' p'}}{\rho} + \frac{\overline{u_i' u_i' u_j'}}{2}\right]}_{\text{turbulent Diffusion}} \underbrace{+ \nu \frac{\partial^2 k}{\partial x_j \partial x_j}}_{\text{Viscous Dissipation}} \underbrace{- \nu \overline{\frac{\partial u_i'}{\partial x_j}\frac{\partial u_i'}{\partial x_j}}}_{\text{Dissipation}}$$

Orders of magnitude: u^3/Λ (convection), u^3/Λ (production), u^3/Λ (turbulent diffusion), $\nu u^2/\Lambda^2$ (viscous dissipation), $\nu \upsilon^2/\eta^2$ (dissipation)

Energy Equation in a Pure Shear Flow

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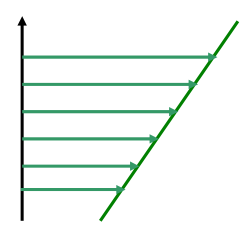
Turbulence Kinetic Energy Equation

$$\frac{D}{Dt}\frac{\overline{u_i' u_i'}}{2} = -\frac{\partial}{\partial x_j}\left[\overline{u_j'\left(\frac{p'}{\rho} + \frac{u_i' u_i'}{2}\right)}\right] - \overline{u_i' u_j'}\frac{\partial U_i}{\partial x_j} - \varepsilon + \overline{\frac{p'}{\rho}\frac{\partial u_j'}{\partial x_j}}$$

$$\mathbf{U} = (U_1(y), 0, 0)$$

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial z} = \mathbf{U}\cdot\nabla = 0$$



Energy Equation in a Pure Shear Flow

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$$0 = -\frac{\partial}{\partial y}\left[\overline{u_2'\left(\frac{p'}{\rho} + \frac{u_i' u_i'}{2}\right)}\right] - \overline{u_1' u_2'}\frac{\partial U_1}{\partial y} - \varepsilon + \overline{\frac{p'}{\rho}\frac{\partial u_j'}{\partial x_j'}}$$

$\frac{1}{2}\overline{u_1'^2}$ ⇒

$$0 = -\frac{\partial}{\partial y}\left(\overline{u_2'\frac{u_1' u_1'}{2}}\right) - \overline{u_1' u_2'}\frac{\partial U_1}{\partial y} - \frac{1}{3}\varepsilon + \overline{\frac{p'}{\rho}\frac{\partial u_1'}{\partial x}}$$

$\frac{1}{2}\overline{u_2'^2}$ ⇒

$$0 = -\frac{\partial}{\partial y}\left[\overline{u_2'\left(\frac{p'}{\rho} + \frac{u_2' u_2'}{2}\right)}\right] - \frac{1}{3}\varepsilon + \overline{\frac{p'}{\rho}\frac{\partial u_2'}{\partial y}}$$

$\frac{1}{2}\overline{u_3'^2}$ ⇒

$$0 = -\frac{\partial}{\partial y}\left[\overline{u_2'\left(\frac{u_3' u_3'}{2}\right)}\right] - \frac{1}{3}\varepsilon + \overline{\frac{p'}{\rho}\frac{\partial u_3'}{\partial z}}$$