

Features of Turbulence

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Outline

- ▶ Definition of Turbulence
- ▶ Features of Turbulence
- ▶ Kolmogorov Scales
- ▶ Energy Cascade

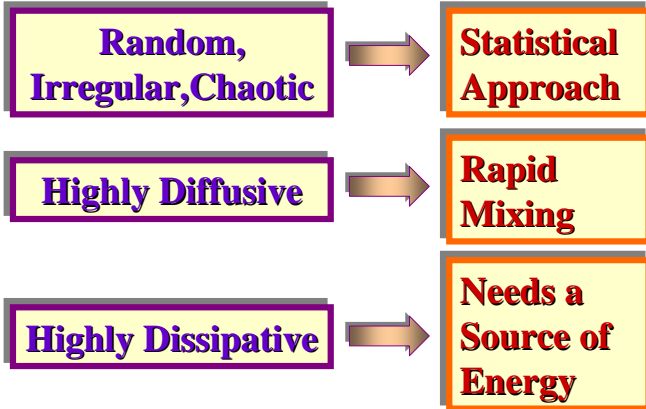
G.I. Taylor & von Karman (1937)

“Turbulence is an irregular motion which in general makes its appearance in fluids, gaseous or liquid, when they flow past solid surfaces or even when neighboring streams of the same fluid flow past or over one another.”

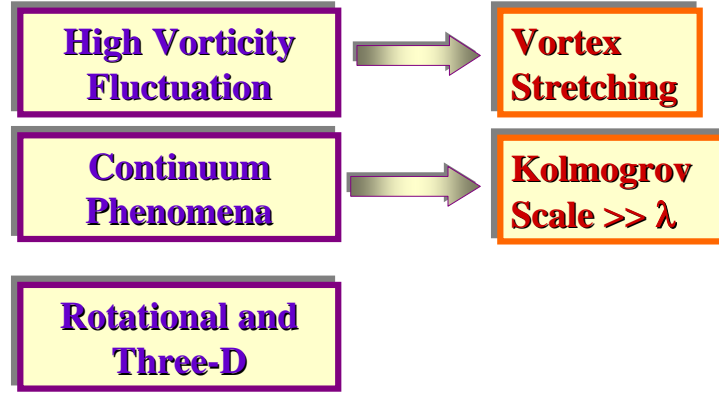
Hinze (1959)

“Turbulent fluid motion is an irregular condition of flow in which the various quantities show a random variation with time and space coordinates, so that statistically distinct average values can be discerned.”

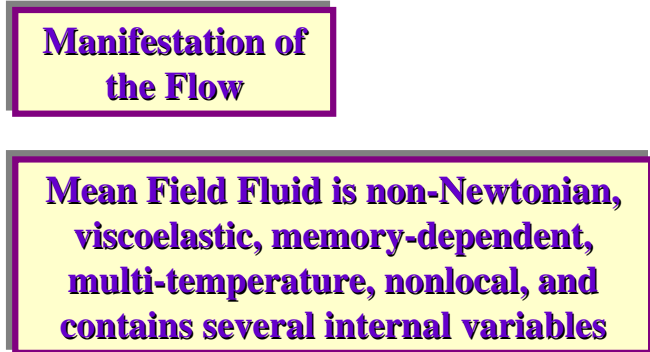
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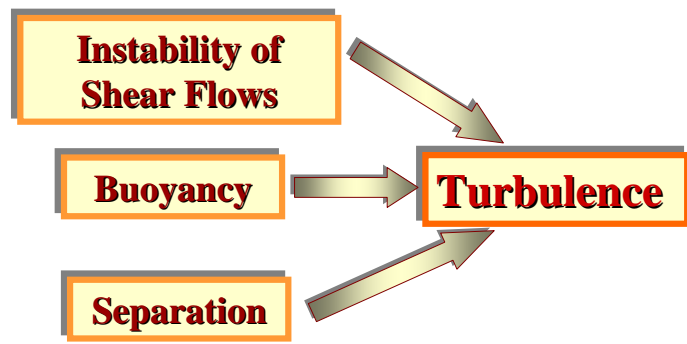
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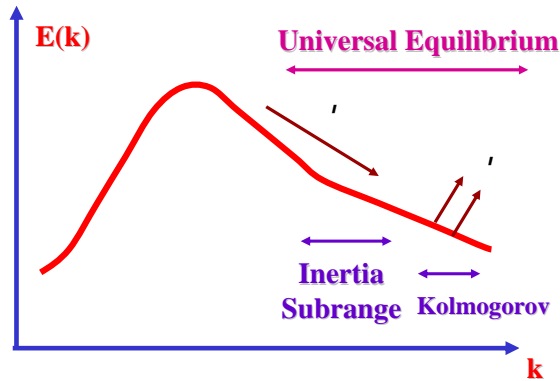
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Origin of Turbulence



Energy Spectrum of Turbulence Clarkson University



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Turbulence Dissipation Clarkson University

Dissipation \cong Production

$$\varepsilon = \frac{u^3}{\Lambda}$$

Direct Viscous Dissipation

$$\nu \left(\frac{\partial U}{\partial y} \right)^2 \sim \nu \frac{u^2}{\Lambda^2}$$

$$\frac{\nu \left(\frac{\partial U}{\partial y} \right)^2}{\varepsilon} = \frac{\text{Large Eddy Direct Viscous Dissip.}}{\text{Turbulence Dissipation Rate}} = \frac{\nu \frac{u^2}{\Lambda^2}}{\frac{u^3}{\Lambda}} = \frac{1}{\text{Re}_\Lambda}$$

Reynolds Number

$$\text{Re}_\Lambda = \frac{u\Lambda}{\nu}$$

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Kolmogorov Scales Clarkson University

Turbulent Motions

- Large-scale turbulent motion is roughly independent of viscosity.
- The small-scale motion is controlled by viscosity.
- Small-scale motions are statistically independent of large-scale turbulent fluctuations (and/or mean motions).

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Kolmogorov Scales Clarkson University

Kolmogorov (Universal Equilibrium Theory)

Small-scale turbulence is in equilibrium (independent of large-scale) and is controlled solely with dissipation rate, ε , and viscosity, ν .

Length Scale

$$\eta \equiv \left(\frac{\nu^3}{\varepsilon} \right)^{1/4}$$

Time Scale

$$\tau \equiv \left(\frac{\nu}{\varepsilon} \right)^{1/2}$$

Velocity Scale

$$U \equiv (\nu\varepsilon)^{1/4}$$

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Length Scale

$$\frac{\eta}{\Lambda} \sim Re_{\Lambda}^{-\frac{3}{4}}$$

Time Scale

$$\frac{\tau U}{\Lambda} \sim Re_{\Lambda}^{-\frac{1}{2}}$$

Velocity Scale

$$\frac{v}{U} \sim Re_{\Lambda}^{-\frac{1}{4}}$$

For Water

$$\varepsilon = 1 \frac{W}{kg} \longrightarrow \eta \approx 30 \mu m$$

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Kolmogorov Inertia Subrange Spectrum Clarkson University

Kolmogorov -5/3 Law

For eddies much smaller than the energy containing eddies and much larger than η , turbulence is controlled by the dissipation rate, ε , and wave number k .

$$E(k) \sim \frac{v_k^2}{k} \sim \frac{\left[\left(\frac{\varepsilon}{k} \right)^{\frac{1}{3}} \right]^2}{k} \sim \varepsilon^{\frac{2}{3}} k^{-\frac{5}{3}}$$

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Energy Cascade Clarkson University

Richardson Verse

“Big whirls have little whirls that feed on their velocity. Little whirls have lesser whirls, and so on to viscosity.”

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Estimates of Turbulence Scales Clarkson University

For a Pipe of Diameter d

Energy containing Eddies

$$\ell_e = 0.05d Re^{-\frac{1}{8}}$$

$$f_e \sim \frac{u^*}{\ell_e} = 20 \frac{u^*}{d} Re^{\frac{1}{8}} \approx 4 \frac{\bar{U}}{d}$$

Kolmogorov Eddies

$$\eta = 4d Re^{-0.78}$$

$$f_k = \frac{v}{\eta} = 0.06 \frac{\bar{U}}{d} Re^{0.56} = \frac{17u^{*2}/v}{Re^{0.44}}$$

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Estimates of Turbulence Scales Clarkson University

Most dissipative Eddies (5η)

$$\ell_d = 20d \text{Re}^{-0.78}$$

$$f_d = 0.02 \frac{\bar{U}}{d} \text{Re}^{0.56} = \frac{6u^{*2}/\nu}{\text{Re}^{0.44}}$$

Largest Eddies

$$d/2$$

$$f_L = \frac{2u^*}{d} = 0.4 \frac{\bar{U}}{d} \text{Re}^{-\frac{1}{8}}$$

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Example Clarkson University

For a pipe with $d=5\text{cm}$, $V=1.8\text{m/s}$ and $\text{Re}=10^5$

Table of eddy size and frequencies.

Eddies	Size	Frequency
Largest Eddies	25 mm	3.5 Hz
Energy Containing Eddies	0.6 mm	140 Hz
Most Dissipative Eddies	0.125 mm	450 Hz
Kolmogorov Eddies	0.025 mm	1300 Hz

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Example- Kolmogorov Scale in Inertial Sublayer Clarkson University

$$-\overline{u'v'} \approx u^{*2}$$

$$\frac{\partial U}{\partial y} \approx \frac{u^*}{\kappa y}$$

$$\text{Production} = -\overline{u'v'} \frac{\partial U}{\partial y} = \frac{u^{*3}}{\kappa y}$$

$$\varepsilon = \frac{u^{*3}}{\kappa y}$$

Kolmogorov Length Scale

$$\eta = \left(\frac{\nu^3}{\varepsilon} \right)^{\frac{1}{4}}$$

$$\eta^+ = \frac{\eta u^*}{\nu}$$

$$\eta^+ = (\kappa y^+)^{\frac{1}{4}}$$

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Kolmogorov Scale in Inertial Sublayer Clarkson University

Turbulence Macroscale

$$\Lambda = \kappa y$$

$$\Lambda^+ = \frac{\Lambda u^*}{\nu} = \kappa y^+$$

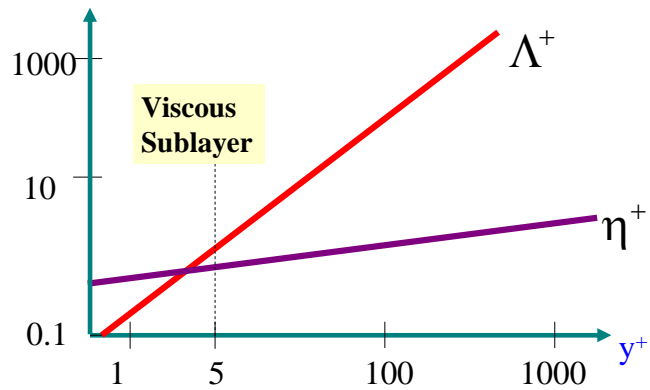
y^+	$\eta^+ = (\kappa y^+)^{\frac{1}{4}}$	$\Lambda^+ = \kappa y^+$
5	12	2
12	15	4
40	2	16
200	3	80
1000	45	400

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Turbulence Scales

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Concluding Remarks

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