TURBULENCE

Features of Turbulence

i) Turbulence is irregular and seemingly random (chaotic). Statistical methods should be used for extracting useful engineering information.

ii) Turbulence is highly diffusive. Rapid mixing significantly increases momentum, heat, and mass transfer.

iii) Turbulence is rotational and three-dimensional motion.

iv) Turbulence is associated with high levels of vorticity fluctuation. Smaller scales are generated by the vortex stretching mechanism.

v) Turbulence is highly dissipative. It needs a source of energy to be maintained.

vi) Turbulence is a continuum phenomenon. The smallest scale of turbulence is much larger than the molecular scales in most engineering applications.

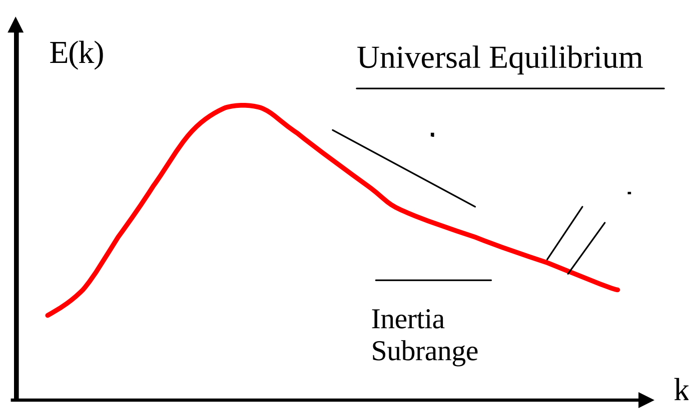
vii) Turbulence is a manifestation of flow and not of the fluid.

viii) The mean field fluid is non-Newtonian, viscoelastic, memory-dependent, multi-temperature, nonlocal, and contains several internal variables.

Origin of Turbulence

Turbulence is associated with high Reynolds number. Its origin is rooted in the instability of shear flows. Turbulence is also generated in buoyancy driven flows.

Energy Spectrum of Turbulence



Schematics of turbulence energy spectrum.

Turbulence has a wide range of length (time) scales. A typical energy spectrum (Fourier decomposition of energy) is shown in the figure. Here E(k) is the energy spectrum and k is wave number (inverse wavelength $(1/\ell)$). Fluctuation energy is produced at the large eddies (with low wave numbers). Vortex stretching mechanism then generates smaller and smaller eddies and energy flows down the spectrum to high wave number region. The energy is mainly dissipated into heat at the smallest eddies (of the order of the Kolmogorov scales).

The dissipation rate, ε, is roughly equal to the fluctuation energy production rate. Suppose the large-scale velocity fluctuation of turbulence is u and the corresponding length scale is Λ. Then the rate of production (or dissipation) of fluctuation energy is given by

$$\varepsilon = \frac{u^3}{\Lambda}. \tag{1}$$

Equation (1) implies that large eddies lose a significant fraction of their energy in a time period of $\frac{\Lambda}{u}$. Note that the direct viscous dissipation rate is

$$\upsilon\left(\frac{\partial U}{\partial y}\right)^2 \sim \upsilon\frac{u^2}{\Lambda^2} \tag{2}$$

and the ratio

$$\frac{\upsilon\left(\frac{\partial U}{\partial y}\right)^2}{\varepsilon} = \frac{\text{Large Eddy Direct Viscous Dissip.}}{\text{Turbulence Dissipation Rate}} = \frac{\upsilon\frac{u^2}{\Lambda^2}}{\frac{u^3}{\Lambda}} = \frac{1}{\text{Re}_\Lambda}, \tag{3}$$

where

$$\text{Re}_\Lambda = \frac{u\Lambda}{\nu} \tag{4}$$

is a characteristic Reynolds number.

Kolmogorov Scales

Large-scale turbulent motion is roughly independent of viscosity. The small-scale, however, is controlled by viscosity. The small-scale motions are also statistically independent of relative slow large-scale turbulent fluctuations (and/or mean motions). According to Kolmogorov (Universal Equilibrium Theory), the small-scale turbulence is in equilibrium (independent of large-scale) and is controlled solely with ε and ν. Using dimensional arguments, Kolmogorov defined the length, time, and velocity scales of the smallest eddies of turbulence. These are

$$\eta \equiv \left(\frac{\nu^3}{\varepsilon}\right)^{\frac{1}{4}}, \ \tau \equiv \left(\frac{\nu}{\varepsilon}\right)^{\frac{1}{2}}, \ \upsilon \equiv (\nu\varepsilon)^{\frac{1}{4}}. \tag{5}$$

Using equation (1), from (5) it follows that

$$\frac{\eta}{\Lambda} \sim \mathrm{Re}_{\Lambda}^{-\frac{3}{4}}, \ \frac{\tau U}{\Lambda} \sim \mathrm{Re}_{\Lambda}^{-\frac{1}{2}}, \ \frac{\upsilon}{U} \sim \mathrm{Re}_{\Lambda}^{-\frac{1}{4}}. \tag{7}$$

For a dissipation rate of $1\frac{W}{kg}$ of water, $\eta \approx 30\mu m$.

Kolmogoroc Inertia Subrange Spectrum

For eddies much smaller than the energy containing eddies and much larger than dissipative eddies (of the order of Kolmogorov scales), turbulence is controlled solely by the dissipation rate ε and the size of the eddy ($\frac{1}{k}$). In this subrange,

$$E(k) \sim \frac{\upsilon_k^2}{k} \sim \frac{\left[(\frac{\varepsilon}{k})^{\frac{1}{3}}\right]^2}{k} \sim \varepsilon^{\frac{2}{3}} k^{-\frac{5}{3}} \tag{8}$$

which is the famous -5/3 law of Kolmogorov. In the derivation of (8), simple dimensional argument is used and velocity scale of eddies of size ($\frac{1}{k}$), $\upsilon_k \sim \left(\frac{\varepsilon}{k}\right)^{\frac{1}{3}}$ is used.

The process of energy cascade in turbulence is summarized by Richardson verse:

"*Big whirls have little whirls that feed on their velocity. Little whirls have lesser whirls, and so on to viscosity.*"

Eddy Frequency Range

For a pipe 2 cm in diameter with water flowing at about 0.5 m/s ($\mathrm{Re} \sim 10^4$), the energy containing eddies have a frequency of about 10-20 Hz. The dissipative eddies are in the range of 100-300 Hz. For $\mathrm{Re} \sim 10^5$, the dissipative eddies get into range of kHz.

The size of large eddies in the core region is about 50% of the pipe diameter (i.e. about 1 cm).

Approximate Estimates for Scales of Turbulence

For a pipe of diameter d, the energy containing eddies are of the order of

$$\ell_e = 0.05 d \, Re^{-\frac{1}{8}} \tag{9}$$

with a frequency of

$$f_e \simeq \frac{u^*}{\ell_e} = 20 \frac{u^*}{d} Re^{\frac{1}{8}} \approx 4 \frac{\overline{U}}{d}, \tag{10}$$

where $u^* = \sqrt{\frac{\tau_0}{\rho}}$ is the shear velocity and $\overline{U}$ is the mean velocity in the pipe. The Kolmogorov length scale is given by

$$\eta = 4d \, Re^{-0.78}, \tag{11}$$

and

$$f_k = \frac{\upsilon}{\eta} = 0.06 \frac{\overline{U}}{d} Re^{0.56} = \frac{17 u^{*2} / \nu}{Re^{0.44}} \tag{12}$$

is the frequency of the Kolmogorov eddies.

The size of the most dissipative eddies is (about 5η)

$$\ell_d = 20 d \, Re^{-0.78} \tag{13}$$

with a frequency of ($\frac{1}{3} f_k$)

$$f_d = 0.02 \frac{\overline{U}}{d} Re^{0.56} = \frac{6 u^{*2} / \nu}{Re^{0.44}} \tag{14}$$

The largest eddies in the pipe are of the order of $\frac{d}{2}$ with a frequency of about $f_L = \frac{2u^*}{d} = 0.4\frac{\overline{U}}{d}\mathrm{Re}^{-\frac{1}{8}}$.

Example

For water flowing through a 50 mm diameter pipe at velocity of 1.8 m/s (Reynolds number $\mathrm{Re} \approx 10^5$), the corresponding scales of turbulence are summarized in the table.

Table of eddy size and frequencies.

Eddies	**Size**	**Frequency**
Largest Eddies	25 mm	3.5 Hz
Energy Containing Eddies	0.6 mm	140 Hz
Most Dissipative Eddies	0.125 mm $=125\,\mu\mathrm{m}$	450 Hz
Kolmogorov Eddies	0.025 mm $=25\,\mu\mathrm{m}$	1300 Hz