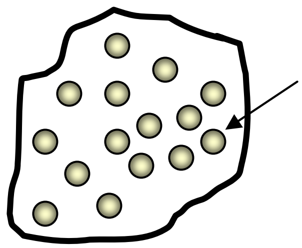
FUNDAMENTALS

Continuum Assumption

A fundamental hypothesis used commonly in fluid mechanics is the continuum assumption. That is we assume that the fluid is a continuum with its physical properties distributed throughout the space. Continuum assumption is valid as long as the smallest dimension of interest is much larger than the molecular scales.

For a continuum the fluid properties are defined as

Density:



3×10^{19} molecules in 1cm^3

$$\rho = \lim_{L\to 0}\frac{\sum m_i}{V}$$

Velocity (mass-averaged):

$$\mathbf{v} = \lim_{L\to 0}\frac{\sum m_i \mathbf{v}_i^{(k)}}{\sum m_i}$$

Molar Averaged Velocity

$$\mathbf{V}^{(k)} = \lim_{L\to 0}\frac{\sum \mathbf{v}_i^{(k)}}{n^{(k)}},$$

where $n^{(k)}$ is the number of molecules of species k and $\mathbf{v}_i^{(k)}$ is the velocity of molecules of species k.

$$\mathbf{V} = \lim_{L\to 0}\frac{\sum \mathbf{v}_i}{n},$$

where n is the number of molecules and $\mathbf{v}_i$ is the velocity. Note that $\mathbf{V} = \mathbf{v}$ if the fluid has a uniform chemical composition.

Fluctuation Velocity

$$\mathbf{v}'_i = \mathbf{v}_i - \mathbf{v}$$

Internal Energy Density

Internal energy density = Fluctuation energy per unit mass = $e = \lim_{L \to 0} \frac{\sum m_i \frac{1}{2} \overline{\mathbf{v}'_i \cdot \mathbf{v}'_i}}{\sum m_i}$

Thermodynamics

- Thermodynamic properties (temperature, entropy, internal energy, enthalpy, etc.) are related.
- For a thermodynamic state, all properties are specified.
- A process constitutes a change in state.
- Reversible process is a sequence of thermodynamical state.
- Extensive properties are proportional to the mass of the system.
- Intensive properties are independent of the mass of the system.

Temperature

$$\frac{3}{2} kT = \frac{1}{2} m \overline{\mathbf{u}'^2} \ ,$$

where T is temperature, $\mathbf{u}'^2$ is kinetic fluctuation energy of molecules, m is the mass of the molecule, and k is the Boltzmann constant.

Entropy

Entropy measures the irreversibility of the process. For reversible processes, in the absence of heat transfer, entropy is constant and increases for irreversible processes. For reversible processes,

$ds = \frac{dQ}{T}$, where s is entropy and Q is heat transfer.

For a system of particles,

$s = k \ln f$, where f is the probability density function.

Basic Equations

$$Tds = de + pd\vartheta$$

$$de = Tds - pd\vartheta = Tds + \frac{p}{\rho^2}d\rho \qquad ,$$

$$e = e(s,\vartheta) \quad de = \frac{\partial e}{\partial s}ds + \frac{\partial e}{\partial \vartheta}d\vartheta = \frac{\partial e}{\partial s}ds + \frac{\partial e}{\partial \rho}d\rho$$

where p is the pressure, $\vartheta = \frac{1}{\rho}$ is the specific volume, and e is the internal energy per unit mass. Thus,

$$T = \left.\frac{\partial e}{\partial s}\right|_{\vartheta}, \; p = -\left.\frac{\partial e}{\partial \vartheta}\right|_{s} = \rho^2 \left.\frac{\partial e}{\partial \rho}\right|_{s}$$

Helmholtz Free Energy Function

$$\psi = e - Ts$$

$$d\psi = de - Tds - sdT = -sdT + \frac{p}{\rho^2}d\rho$$

For $\psi = \psi(T,\rho)$,

$$d\psi = \frac{\partial \psi}{\partial T}dT + \frac{\partial \psi}{\partial \rho}d\rho$$

Hence,

$$s = -\left.\frac{\partial \psi}{\partial T}\right|_{p}, \; p = \rho^2 \left.\frac{\partial \psi}{\partial \rho}\right|_{T}$$

Enthalpy $h = e + \frac{p}{\rho}$

Isothermal compressibility coefficient: $\alpha = \frac{1}{\rho}\left.\frac{\partial \rho}{\partial P}\right|_{T}$

Bulk expansion coefficient: $\beta = \frac{1}{\rho}\left.\frac{\partial \rho}{\partial T}\right|_{p}$

Ideal Gas

$$p = \rho R T \, , \; h = e + RT$$

$$c_P = \left.\frac{\partial h}{\partial T}\right|_P = c_V + R \, , \; c_V = \left.\frac{\partial e}{\partial T}\right|_P \, , \quad \gamma = \frac{c_P}{c_V}$$

For an incompressible substance:

$$c_P = c_V \, , \; \gamma = 1$$

Compressibility Factor

$$Z = \frac{p}{\rho R T}$$