INDICIAL NOTATION

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Goodarz Ahmadi
Department of Mechanical and Aeronautical Engineering
Clarkson University, Potsdam, NY 13699-5725

Outline

- Basic Rules
- Vectors and Tensors
- Tensor Operation
- Isotropic Tensors

INDICIAL NOTATION

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(Cartesian Tensor)

Basic Rules

- A free index appears only once in each term of a tensor equation. The equation then holds for all possible values of that index.
- Summation is implied on an index, which appears twice.
- No index can appear more than twice in any term.

Definition (Cartesian Tensors)

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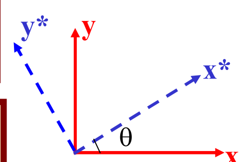
Change of Frame

$$x_i^* = Q_{ij}x_j$$

$$x_j = Q_{ij}x_i^*$$



$$\det\left|Q_{ij}\right| = \pm 1$$

$$Q_{ij}Q_{ik} = \delta_{jk} \qquad Q_{ij}Q_{kj} = \delta_{ik}$$

Definition (Cartesian Tensors)

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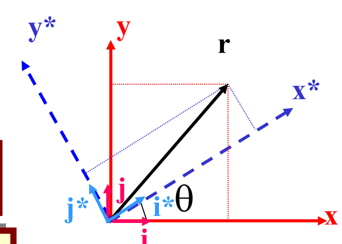
$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} = x^*\mathbf{i}^* + y^*\mathbf{j}^*$$

$$\mathbf{i}^* = \mathbf{i}\cos\theta + \mathbf{j}\sin\theta$$

$$\mathbf{j}^* = -\mathbf{i}\sin\theta + \mathbf{j}\cos\theta$$

$$x^* = x\cos\theta + y\sin\theta$$

$$y^* = -x\sin\theta + y\cos\theta$$



Definition (Cartesian Tensors)

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Transformation in Two Dimension

$$[Q]=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$$

Kronecker Delta → $$[\delta_{ij}]=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

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Definition (Cartesian Tensors)

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Scalar → $T^* = T$

Vector → $\mathbf{v}^* = \mathbf{Q}\cdot\mathbf{v}$

Second Order Tensor → $\boldsymbol{\tau}^* = \mathbf{Q}\cdot\mathbf{t}\cdot\mathbf{Q}^T$

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Vectors and Tensors

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Vector → $v_i^* = Q_{ij} v_j$

Second Order Tensor → $t_{ij}^* = Q_{ik} Q_{jl} t_{kl}$

Third Order Tensor → $\lambda_{ijk}^* = Q_{im} Q_{jn} Q_{kl} \lambda_{mnl}$

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Definition (Cartesian Tensors)

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Alternating Symbol → ε_{ijk}

$\varepsilon_{ijk} = 1$, for i, j, k even permutation

$\varepsilon_{ijk} = -1$, for i, j, k odd permutation

$\varepsilon_{ijk} = 0$, when two indices are equal

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Tensor Operations

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Gradient

$$(\nabla\varphi)_i = \frac{\partial \varphi}{\partial x_i} = \varphi_{,i}$$

$$(\nabla \mathbf{v})_{ij} = \frac{\partial v_j}{\partial x_i} = v_{j,i}$$

Divergence

$$\nabla \cdot \mathbf{v} = v_{i,i}$$

$$(\nabla \cdot \boldsymbol{\tau})_j = \frac{\partial \tau_{ij}}{\partial x_i} = \tau_{ij,i}$$

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Tensor Operations

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Curl

$$(\nabla \times \mathbf{U})_i = \varepsilon_{ijk} \frac{\partial U_k}{\partial x_j} = \varepsilon_{ijk} U_{k,j}$$

Determinant

$$\det|\mathbf{A}| = \varepsilon_{ijk} A_{1i} A_{2j} A_{3k}$$

Identity

$$\varepsilon_{ijk}\varepsilon_{imn} = \delta_{jm}\delta_{kn} - \delta_{jn}\delta_{km}$$

Laplacian

$$\nabla^2 \varphi = \frac{\partial^2 \varphi}{\partial x_i \partial x_i} = \varphi_{,ii}$$

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Isotropic Tensors

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Rank Zero: → **All Scalars**

Rank One: → **None**

Rank Two: → $\alpha\delta_{ij}$

Rank Three: → $\alpha\varepsilon_{ijk}$

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Isotropic Tensors

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Rank Four:

$$\alpha\delta_{ij}\delta_{kl} + \beta(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) + \gamma(\delta_{ik}\delta_{jl} - \delta_{il}\delta_{jk})$$

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